# Visual Computing Q&A Session

**Autumn Semester 2023**

# WebGL & Graphics Pipeline

In Exercise 6 we have seen that vertex coloring and the computation of normals can also be done outside the shader files. In fact, all the code from the shader files can also be moved to the JS file.
What is the advantage of using shader files?

**Parallelization:** OpenGL/WebGL is optimized to run the shader code in parallel on GPUs
**Portability:** Shader files can be reused independent of how the mesh is read
**Simplicity:** The shader code targets the computation for one instance (vertex or fragment)
**Interpolation:** In the fragment shader we can make use of already interpolated values (due to rasterization)

# WebGL & Graphics Pipeline (2010, 6a)

a) **Graphics Pipeline**

The following diagram shows an overview of the graphics pipeline.

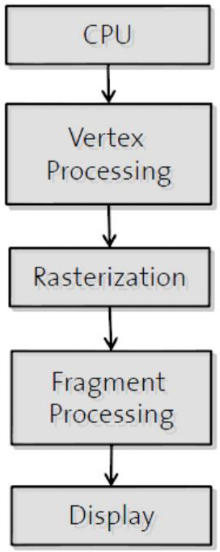
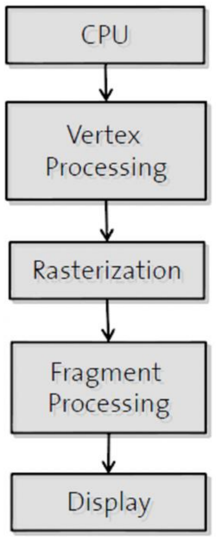



Figure 10: Simplified Graphics Pipeline

# WebGL & Graphics Pipeline (2010, 6a)




i) Referring to the diagram, explain the difference between the fixed function pipeline and the programmable pipeline. **3 pts.**

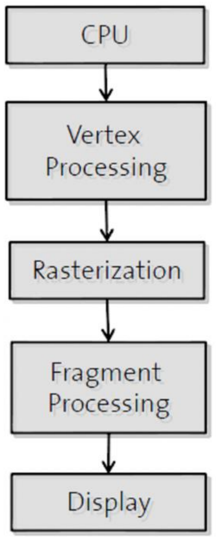
ii) What is the purpose of the rasterization step? What are the inputs and outputs? **3 pts.**

# WebGL & Graphics Pipeline (2010, 6b)

ii) A usual vertex shader contains the line

```
gl_Position = proj_mat * modelview_mat * position;
```

What is computed here? **2 pts.**




iii) In the fragment shader shown above the pixel color is multiplied by an intensity term. What effect does this achieve? **3 pts.**

# Light, Color and Raytracing

## a) CIE Color Spaces

Consider the 3 following primaries given in the CIE xyY color space:

| | x | y | Y |
|---|---|---|---|
| $c_1$ | 0.2 | 0.6 | 12 |
| $c_2$ | 0.2 | 0.05 | 10 |
| $c_3$ | 0.6 | 0.3 | 7 |

i) Explain briefly the perceptual meaning of the xy and Y axes in the CIE xyY color space. **1 pt.**

*xy axes control Chromaticity (color, colorness) and Y controls Luminance (brightness, intensity).*

ii) Provide the transformation formulas from CIE XYZ to CIE xyY and from CIE xyY to CIE XYZ. **2 pts.**

*XYZ to xyY :*

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

*xyY to XYZ :*

$$X = x\frac{Y}{y}$$

$$Z = \frac{Y}{y} - x\frac{Y}{y} - Y$$

*1pt for XYZ to xyY, 1pt for xyY to XYZ.*

iii) Compute the sum of the three primaries $c_{123} = c_1 + c_2 + c_3$ in XYZ color space. What are the coordinates of $c_{123}$ on the CIE xy Chromaticity diagram? **3 pts.**

*From the previous question we can compute X and Z for $c_1$, $c_2$, $c_3$, we get :*
$c_1$ : $X_1 = 4$; $Z_1 = 4$
$c_2$ : $X_2 = 40$; $Z_2 = 150$
$c_3$ : $X_3 = 14$; $Z_3 = 2.333(\frac{7}{3})$
*The addition in XYZ space gives us :*
$X_{123} = X_1 + X_2 + X_3 = 58$
$Y_{123} = Y_1 + Y_2 + Y_3 = 29$
$Z_{123} = Z_1 + Z_2 + Z_3 = 156.333(\frac{469}{3})$
*From the previous question we convert these results to xyY and we get*
$x = 0.238(\frac{87}{365})$, $y = 0.119(\frac{87}{730})$, $Y = 29$

*So the coordinate of $c_{123}$ on the CIE xy Chromaticity diagram is* $(0.238, 0.119)$.

# Light, Color and Raytracing

You would like to render a set of pool balls using raytracing, so you set up scene by placing a camera and multiple spheres that represents your pool balls. To generate your image, you then shoot a ray for each pixel and check for an intersection with the spheres.

i) Assuming the camera is placed at $\mathbf{c} = (0, 0, 10)$, and you shoot a ray defined by the camera position and the direction $\mathbf{d} = (1, 2, -5)$, write down the equation that defines the intersection points between the ray and a sphere with radius $5$ placed at $\mathbf{s} = (3, 3, -2)$. The parameter of the ray equation should be the unknown in your equation. **4 pts.**

# Solution

1. Points on the ray **x** = **r**(t) = **c** + t **d**
2. Point **x** on the sphere satisfy || **x** - **s** ||$^2$ - r$^2$ = 0
3. We want to solve for a t, such that

$$\| \mathbf{c} + t\,\mathbf{d} - \mathbf{s} \|^2 - r^2 = 0$$

$$\left| \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix} \right|^2 - 25 = \left| \begin{pmatrix} -3 \\ -3 \\ 12 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} \right|^2 - 25 = 0$$

# Light, Color and Raytracing

ii) The pool balls that you want to render are partly reflective. Explain briefly how reflections can be incorporated into a raytracer. **3 pts.**

*Shoot a reflected ray from every intersection point, continue raytracing.*

# Light, Color and Raytracing

iii) You realize that your renderer is slow, and the problem seems to be that for every ray, you compute an intersection with every pool ball. Explain briefly how you can avoid computing unnecessary intersections and thus accelerate your renderer, and what steps before and during the rendering you have to add. **4 pts.**

*Use accelerating data structure (e.g., uniform grids)*
*Before rendering: Initalize grid/data structure, insert objects*
*During rendering: Traverse data structure along ray, search for occupied cells*

# Curves & Surfaces

i) Please select the correct statement(s) regarding Bézier and B-Spline curves. You don't need to justify your choice. There are no negative points. **1 pt.**

- ☐ Bézier curves are special cases of B-spline curves.
- ☐ Bézier curves have local support and B-Splines have global support.
- ☐ Insertion of new control points in Bézier curves comes along with degree elevation.

ii) Please select the correct statement(s) regarding subdivision surfaces. Please select the correct statement(s) regarding texture mapping. You don't need to justify your choice. There are no negative points. **1 pt.**

- ☐ The mesh surface converges to smooth limit surface.
- ☐ Subdivision surfaces are the most popular geometry representation in computer aided design for car industry.
- ☐ Loop subdivision scheme works for any polygonal meshes.

# Solution

i) Please select the correct statement(s) regarding Bézier and B-Spline curves. You don't need to justify your choice. There are no negative points. **1 pt.**

- [x] Bézier curves are special cases of B-spline curves.
- [ ] Bézier curves have local support and B-Splines have global support.
- [x] Insertion of new control points in Bézier curves comes along with degree elevation.

ii) Please select the correct statement(s) regarding subdivision surfaces. Please select the correct statement(s) regarding texture mapping. You don't need to justify your choice. There are no negative points. **1 pt.**

- [x] The mesh surface converges to smooth limit surface.
- [ ] Subdivision surfaces are the most popular geometry representation in computer aided design for car industry.
- [ ] Loop subdivision scheme works for any polygonal meshes.

# Geometry and Textures (2020, 7b, iv)

How do we know that the following image was more likely generated using texture mapping and not using a high resolution mesh? **2 pts.**

***The shadow does not have jaggy contours.***

# Geometry and Textures (2010, 6d)

i) If we read out a cube map at direction $\mathbf{d} = (2, 3, 5)^T$, which texture (face) of the cube map will be accessed, and at which texture coordinate? **3 pts.**

*The biggest component is $d_z = 5$, so the positive $z$ face will be accessed. Divide by 5: $(2/5, 3/5, 1)^T$, so coordinates in $[-1, 1]$ are $(2/5, 3/5)^T$. Transform to $[0, 1]$ gives $(u, v)^T = (2/10 + 1/2, 3/10 + 1/2)^T = (0.7, 0.8)^T$.*

# Lighting and Shading (2020, 9b)

Consider the illumination model defined by the following equation

$$I = \underbrace{k_0 I_0}_{\textit{Component 1}} + \underbrace{k_1 I_1 N \cdot L}_{\textit{Component 2}} + \underbrace{k_2 I_2 (V \cdot R)^{\alpha}}_{\textit{Component 3}} \tag{2}$$

where $I_0, I_1, I_2$ denotes the color intensity of these three components respectively, $k_0, k_1, k_2$ are the coefficients of these three terms, $N$ is the surface normal, $L$ is the light direction, $V$ is the viewer direction and $R$ is the reflected light direction.

i) Give the names of components 1, 2 and 3 and the name of this illumination model.

**2pts.**

# Solution (2020, 9b)

*i) Component 1: Ambient*

*Component 2: Diffuse*

*Component 3: Specular*

⇨ *Phong Illumination Model*

# Lighting and Shading (2020, 9b)

ii) One can notice that some geometries in a scene or parts of geometries are **NOT** visible to the light source directly. However, they are not completely dark. Which components(s) from the equation above are responsible for this? Please write down the name(s).

**2pts.**

# Solution (2020, 9b)

ii) One can notice that some geometries in a scene or parts of geometries are **NOT** visible to the light source directly. However, they are not completely dark. Which components(s) from the equation above are responsible for this? Please write down the name(s).

**2pts.**

*The ambient term. The ambient term mimics indirect lighting as one would get from proper path tracing algorithms. The other two terms depend on the light source explicitly as they utilize the light vector L.*

$$I_\lambda = I_{a_\lambda} k_a + f_{att} I_{p\lambda} \left[ k_d(\mathbf{N} \cdot \mathbf{L}) + k_s(\mathbf{R} \cdot \mathbf{V})^n \right]$$

$$\mathbf{R} = \mathbf{N}\cos\theta + \mathbf{S}$$

$$\mathbf{R} = 2\mathbf{N}\cos\theta - \mathbf{L} = 2\mathbf{N}(\mathbf{N} \cdot \mathbf{L}) - \mathbf{L}$$

# Lighting and Shading (2020, 9b)

iii) Explain what visual changes one would expect if the exponent in the given equation increases.

$$I = \underbrace{k_0 I_0}_{\textit{Component 1}} + \underbrace{k_1 I_1 N \cdot L}_{\textit{Component 2}} + \underbrace{k_2 I_2 (V \cdot R)^{\alpha}}_{\textit{Component 3}}$$

# Lighting and Shading (2020, 9b)

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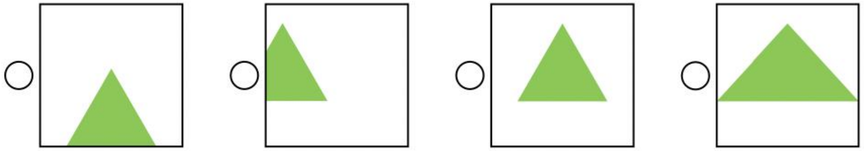
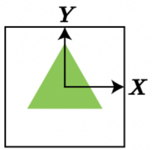
*The specular highlight on the surface appears to be smaller in terms of the area. One could plot the powered cosine function for better understanding. As the power increases, more values are mapped to zero.*

# Transformations (2018, 7b)

i) $\boldsymbol{F}_\theta = \begin{pmatrix} 1 & 0 & \theta \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **4 pts.**



b) Transformation Families



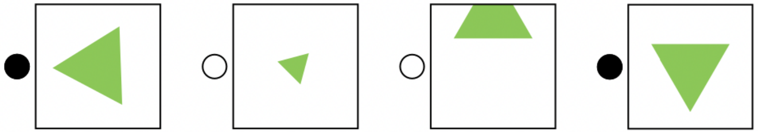

This is the original triangle. The world origin is at the center of the triangle, and the world axes are as drawn as unit vectors. The box around the triangle is **your screen**.

For each question in this section I will give you a parameterized transformation $\boldsymbol{F}_\theta$ and four pictures of triangles **clipped to the screen**. Consider each of the four pictures individually. If a picture is the result of applying $\boldsymbol{F}_\theta$ to the original triangle **for some** $\theta$ then **fill in the circle next to it**.

NOTE: To be clear, if the vertices of the original triangle are $\{\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3\}$, then choose a new triangle if and only if its vertices are $\{\boldsymbol{F}_\theta \boldsymbol{p}_1, \boldsymbol{F}_\theta \boldsymbol{p}_2, \boldsymbol{F}_\theta \boldsymbol{p}_3\}$ for some $\theta$.

NOTE: I'm not trying to trick you with the pictures. If a picture looks like the original triangle moved to the right, then it is the original triangle moved to the right.

**EXAMPLE:** $\boldsymbol{F}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

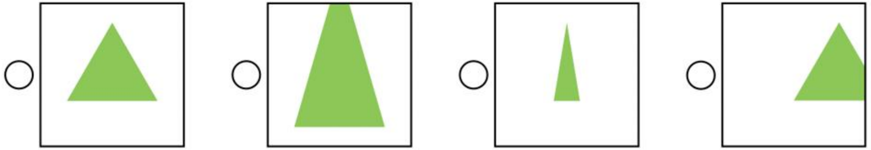


Explanation: This is a rotation by $\theta$. I fill in the circles next to the two pictures that could be made by rotating the original triangle about the origin by some angle $\theta$.

The other two pictures cannot possibly be made by $\boldsymbol{F}_\theta$ so I leave their circles blank.

# Transformations (2018, 7b)

ii) $\boldsymbol{F}_\theta = \begin{pmatrix} \theta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **4 pts.**



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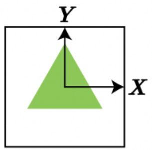


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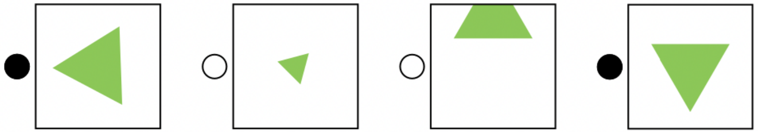

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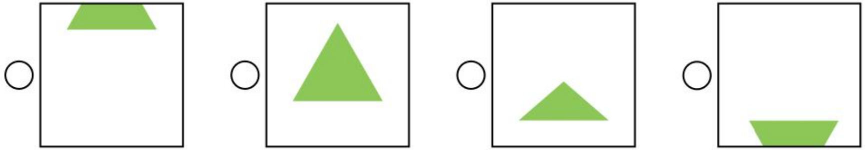
# Transformations (2018, 7b)

iii) (Challenge) $\boldsymbol{F}_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \theta & \theta \\ 0 & 0 & 1 \end{pmatrix}$ **4 pts.**



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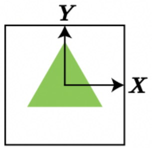


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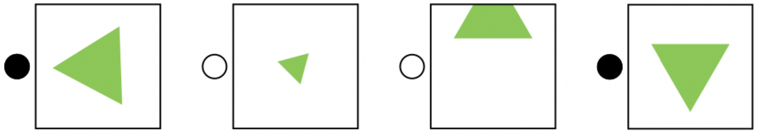

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# Transformations (2016, 9c)

c) **Quaternions**

We are going to perform a transformation $T : \mathbb{R}^{3\times1} \mapsto \mathbb{R}^{3\times1}$ to a point $\mathbf{p}$ using quaternions. The transformation is a rotation around the rotation axis $\mathbf{u} = (1, 1, 0)^T$ with an angle of $\theta = \frac{\pi}{2}$. The point $\mathbf{p}$ is expressed in euclidean coordinates as $\mathbf{p} = (5, 0, 0)^T$. Follow the steps below to find $p'$, the rotated point corresponding to $\mathbf{p}' = T(\mathbf{p})$.

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i) Describe $\mathbf{p}$ in quaternion form. **1 pt.**

ii) Write down the general formula to compute a rotation quaternion $\mathbf{q}_{gen}$ from a rotation axis $\mathbf{u}_{gen}$ and an angle $\theta_{gen}$. Make sure the formula is correct even if $\mathbf{u}_{gen}$ is not a unitary vector. **1 pt.**

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iii) Describe $\mathbf{q}$, a rotation quaternion representing the transformation $T$. Make sure to normalize $\mathbf{u}$ before anything else. **1 pt.**

iv) Write down the general formula to compute $\mathbf{p}'_{gen}$, a transformed point, result of applying a quaternion $\mathbf{q}_{gen}$ to a point $\mathbf{p}_{gen}$. You can assume that $\mathbf{q}_{gen}$ is a unit quaternion. **1 pt.**

# Transformations (2016, 9c)

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v) Now apply the formula that you just wrote in the previous question to compute the rotated point $\mathbf{p}'$. Part of the operation was already done for you and you can use the following quaternion product equality $\mathbf{q} \cdot \mathbf{p} = \frac{1}{2}(-5 + 5\sqrt{2}i - 5k)$ without the need to derive it. **4 pts.**

$$\mathbf{q}\mathbf{p}\bar{\mathbf{q}} = \frac{1}{4}(-5 + 5\sqrt{2}i - 5k)(\sqrt{2} - (i + j)) \tag{1a}$$

$$= \frac{1}{4}(10i - 5\sqrt{2} - 5\sqrt{2}k - 5\sqrt{2}i(i + j) + 5(i + j) + 5k(i + j)) \tag{1b}$$

$$= \frac{1}{4}(10i - 5\sqrt{2} - 5\sqrt{2}k + 5\sqrt{2} - 5\sqrt{2}k + 5i + 5j + 5j - 5i) \tag{1c}$$

$$= \frac{1}{4}(10i + 10j - 10\sqrt{2}k) \tag{1d}$$

$$= \frac{5}{2}i + \frac{5}{2}j - \frac{5}{2}\sqrt{2}k \tag{1e}$$

$$\tag{1f}$$

*Therefore* $\mathbf{p}' = (\frac{5}{2}, \frac{5}{2}, -\frac{5}{2}\sqrt{2})^T$

# Signal Processing

The fourier transform of a signal $g$ is defined as

$$(\mathcal{F}g)(u) = \int_{\mathbb{R}} g(x)\, e^{-2\pi i u x} dx \tag{1}$$

The inverse fourier transform is defined as

$$g(x) = \int_{\mathbb{R}} e^{2\pi i u x} (\mathcal{F}g)(u)\, du \tag{2}$$

The fourier transform of the dirac delta function $\delta(x - x_0)$ is given as.

$$(\mathcal{F}\delta(x - x_0))(u) = e^{-2\pi i u x_0} \tag{3}$$

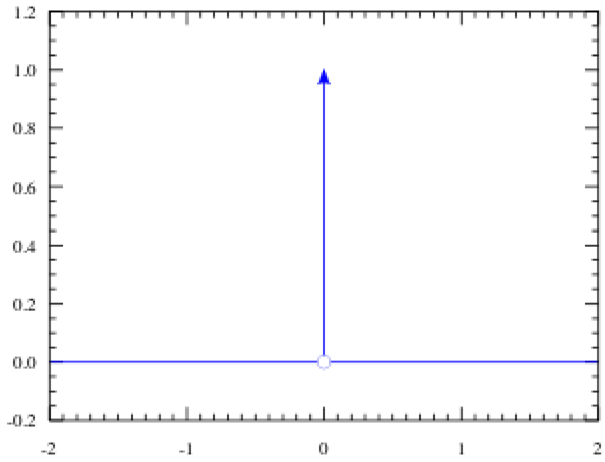
Euler's formula states

$$e^{ix} = \cos(x) + i \sin(x) \tag{4}$$

i) Using Eq. 1 and the properties of the dirac delta function explain why Eq. 3 holds. **3 pts.**

ii) Derive the Fourier Transorm of the signal $f_1(x) = \sin(2\pi x_0 x)$ using the above equations. Your solution must not contain any integrals. **5 pts.**

iii) Given the signal $f_2(x) = e^{-2\pi i x u_0}$ and the fourier transform of $F_3$ of a signal $f_3(x)$. Give the fourier transform $F_4$ of $f_4(x) := f_2(x)f_3(x)$ in terms of $F_3$. **5 pts.**


1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-2
-1
0
1
2

# Solution

1. *Writing out the fourier transform of the dirac delta function we get*

$$(\mathcal{F}\delta\left(x-x_0\right))\left(u\right) = \int_{\mathbb{R}} \delta\left(x-x_0\right) e^{-2\pi i u x} dx$$

By Eqn. (1)

*The dirac delta function is 0 except for* $x = x_0$ *and the integral of the dirac delta function is* $1$*. We end up with*

$$e^{-2\pi i u x_0}$$

2. *Definition of the fourier transform plus euler's formula.*

$$\int_{\mathbb{R}} \sin(2\pi x_0 x) e^{-2\pi i u x} dx = \int_{\mathbb{R}} \frac{e^{2\pi i x_0 x} - e^{-2\pi i x_0 x}}{2i} e^{-2\pi i u x} dx =$$

*Arranging the terms*

$$\frac{1}{2i} \int_{\mathbb{R}} e^{2\pi i(-u)x} e^{-2\pi i(-x_0)x} - e^{2\pi i(-u)x} e^{-2\pi i x_0 x} dx =$$

*Inverse fourier transform of the dirac delta function plus symmetry of the dirac delta function.*

$$\frac{-1}{2i}\left(\delta(-u-x_0) - \delta(-u+x_0)\right) = \frac{-1}{2i}\left(\delta(u+x_0) - \delta(u-x_0)\right)$$

$$r(x) = \{g * h\}(x) \triangleq \int_{-\infty}^{\infty} g(\tau)h(x-\tau)\,d\tau = \int_{-\infty}^{\infty} g(x-\tau)h(\tau)\,d\tau.$$

3. *Multiplication in spatial domain is convolution in frequency domain.*

$$F_4(u) = \int_{\mathbb{R}} e^{-2\pi i x u_0} e^{-2\pi i u x} dx * F_3(u)$$

$$\delta(+u + u_0) * F_3(u) = \delta(u + u_0) * F_3(u) =$$

$$F_3(u + u_0)$$