Tutorial 2

Today's topics:

- Recap about convolutions
- Recap about edge detection
- Coding assignment on edge detection

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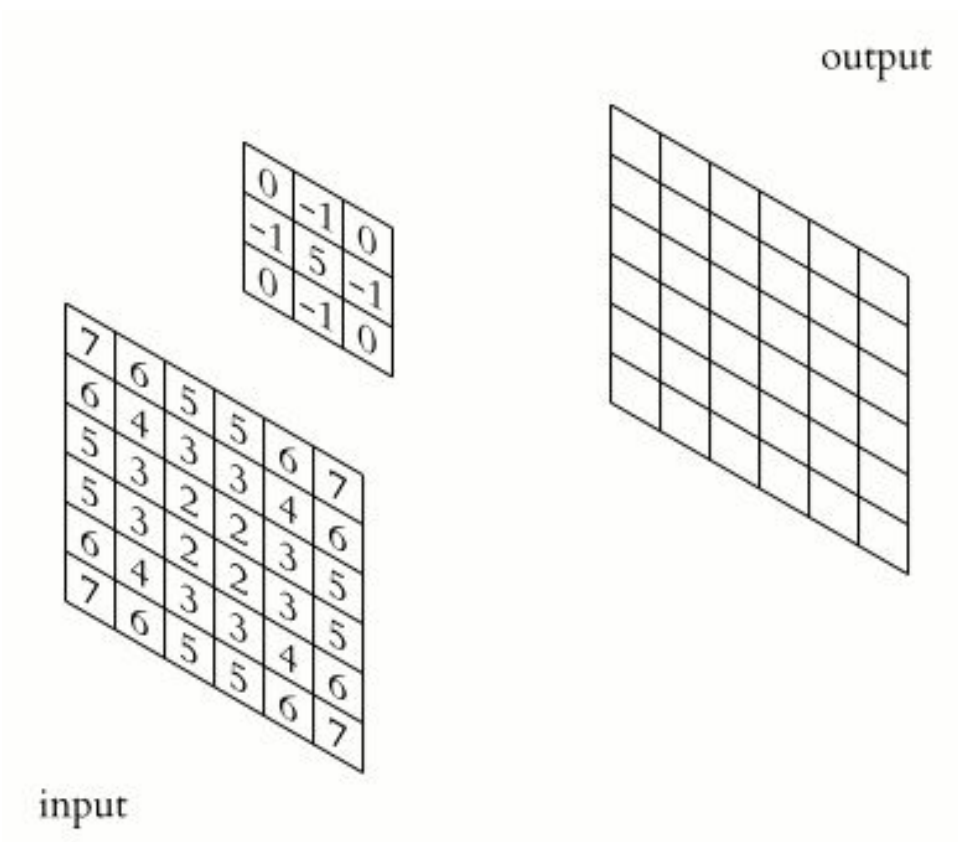
$$I'(x, y) = \sum_{j=-k}^{k} \sum_{i=-k}^{k} K(i, j) I(x - i, y - j)$$

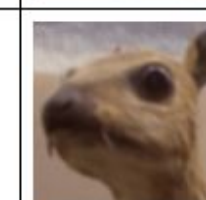
Image Filtering, Convolution

- Image filtered by convolving with a filter kernel
- Convolution denoted by “$*$”

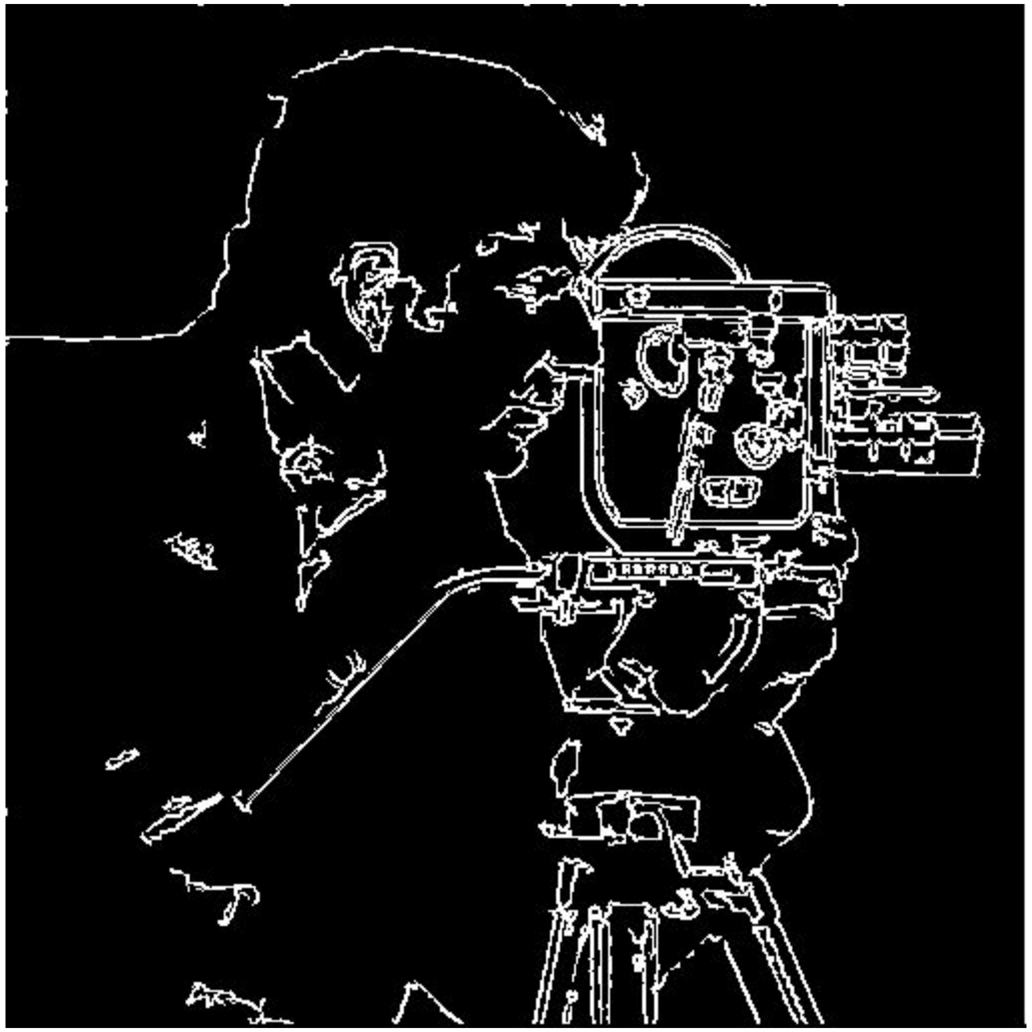
$$I_{output} = k * I_{input}$$



http://en.wikipedia.org/wiki/Kernel_(image_processing)

Original	$\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}$	
Edge-Detect	$\begin{bmatrix}1&0&-1\\0&0&0\\-1&0&1\end{bmatrix}$	
	$\begin{bmatrix}0&1&0\\1&-4&1\\0&1&0\end{bmatrix}$	
	$\begin{bmatrix}-1&-1&-1\\-1&8&-1\\-1&-1&-1\end{bmatrix}$	
Sharpen	$\begin{bmatrix}0&-1&0\\-1&5&-1\\0&-1&0\end{bmatrix}$	
Blur*	$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}$	
	$\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$	

Edge Detection

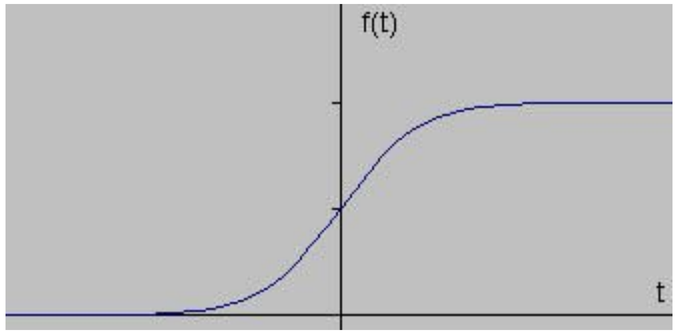


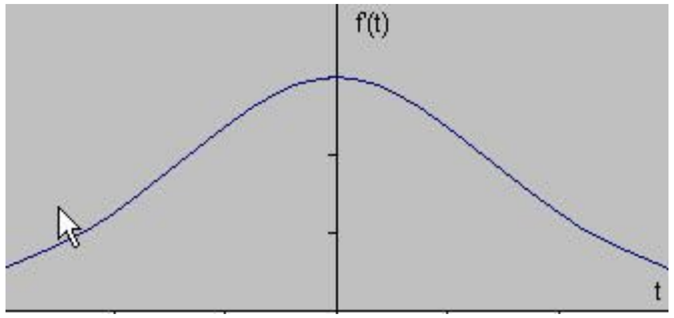
http://vision.cs.arizona.edu/nvs/research/image_analysis/edge.html

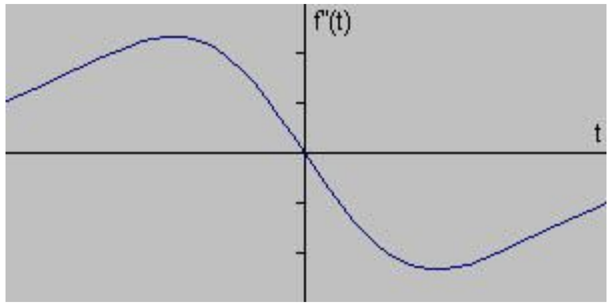
Edges

Edges in images are areas with strong intensity contrasts

- Change is measured by derivative in 1D
- Biggest change, derivative has maximum magnitude
- Or 2[nd] derivative is zero







http://www.pages.drexel.edu/~weg22/edge.html

Gradient Method

Gradient Vector

$$\mathbf{g}(x,y) = \begin{bmatrix} g_x(x,y) \\ g_y(x,y) \end{bmatrix} = \begin{bmatrix} (k_x * f)(x,y) \\ (k_y * f)(x,y) \end{bmatrix}$$

Gradient Magnitude

$$|\mathbf{g}| = \left(g_x^2 + g_y^2\right)^{1/2}$$

Direction

$$\theta = \tan^{-1}\left(\frac{g_y}{g_x}\right)$$

Image gradient?

Usual continuous derivatives:

$$\frac{\partial f}{\partial x} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon, y) - f(x, y)}{\epsilon}$$

Discrete approximation:

$$\frac{\partial f}{\partial x} \approx \frac{f(x+\Delta x, y) - f(x, y)}{\Delta x}$$

Translated in image convolutions:

$$\frac{\partial I}{\partial x} = [-1 \quad 1] * I$$

Sobel kernel

Approximate of the 2D derivative of an image

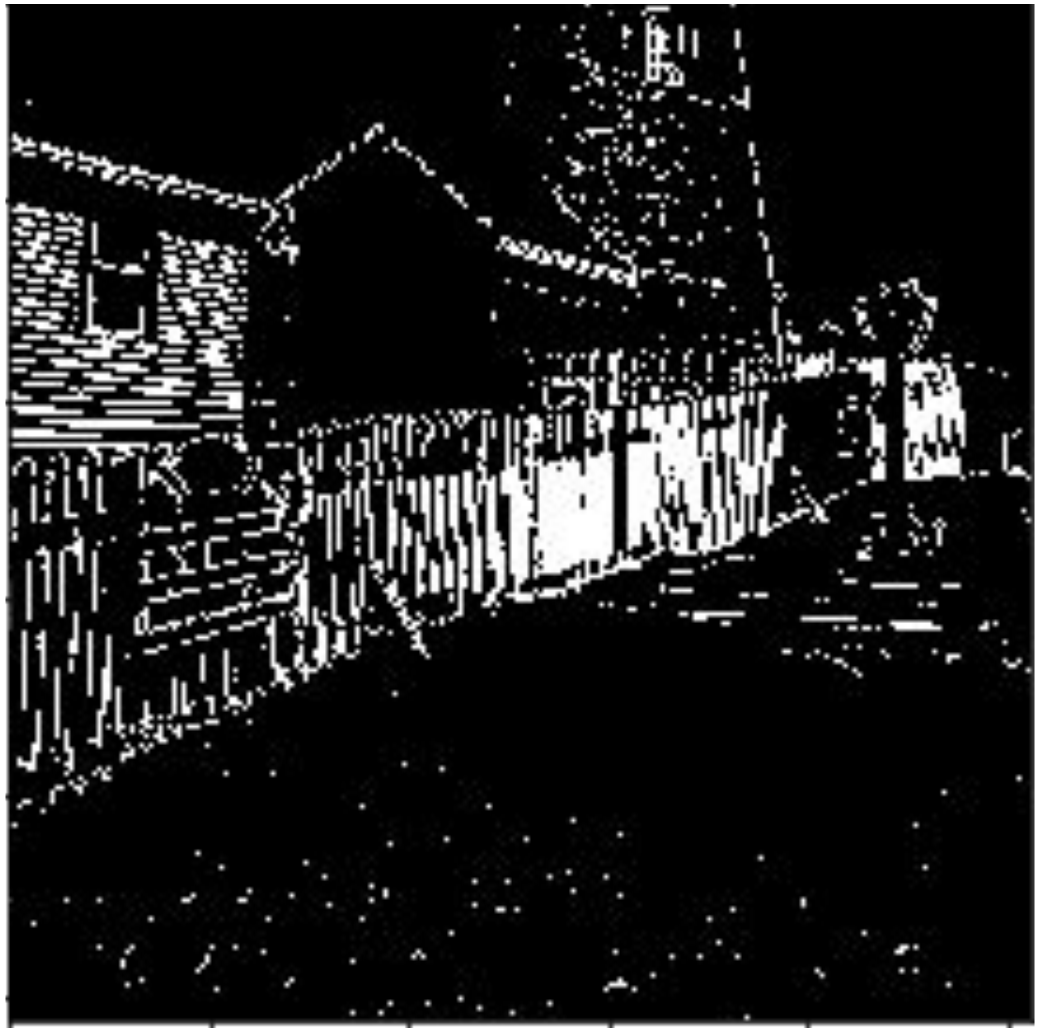
$$k_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad k_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Prewitt kernel

Approximate of the 2D derivative of an image

$$k_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad k_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Gradient Thresholding



Canny Edge Detection

Canny Edge Detection

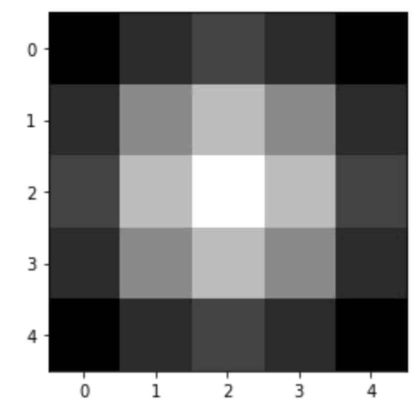
Combine noise reduction and edge enhancement.

1. Apply derivative of Gaussian filter
2. Non-maximum suppression
   - Thin multi-pixel wide “ridges” down to single pixel width
3. Hysteresis
   - Accept all edges over low threshold that are connected to edge over high threshold

Derivative of Gaussian kernel

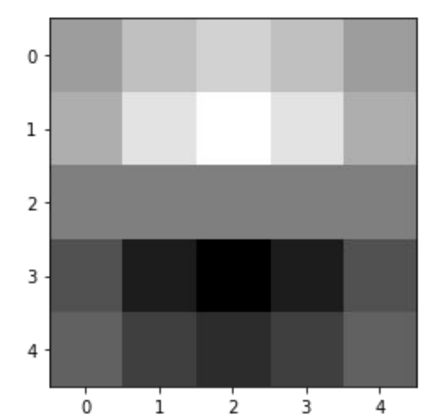
- Need smoothing to reduce noise prior to taking derivative

0.0121	0.0261	0.0337	0.0261	0.0121
0.0261	0.0561	0.0724	0.0561	0.0261
0.0337	0.0724	0.0935	0.0724	0.0337
0.0261	0.0561	0.0724	0.0561	0.0261
0.0121	0.0261	0.0337	0.0261	0.0121



- We can use derivative of Gaussian filters
  - because differentiation is convolution, and convolution is associative:

$$D * (G * I) = (D * G) * I$$



Non-maximum suppression

- The edge direction angle is rounded to one of four angles representing vertical, horizontal and the two diagonals.
- Select the single maximum point across the width of an edge.
  - Maximum: The gradient magnitudes of the two neighbors in edge normal direction are smaller.



courtesy of G. Loy

Hysteresis

Idea: real objects usually define continuous edges, noise is disrupted instead

In practice define two thresholds $T_{low} < T_{high}$ and classify each a gradient pixel G:

- if $G < T_{low}$ then it's definitely ***not* an edge**
- if $G > T_{high}$ then it's definitely a **strong edge**
- if $T_{low} < G < T_{high}$ then it is a **weak edge** if and only if it is connected to any strong edge through other weak edges

Hysteresis

Strong edges only > T_{high}

Weak edges > T_{low}

gap is gone

Strong + connected weak edges

courtesy of G. Loy

Coding assignment this week

Coding assignment available on same repository as last week (https://github.com/tavisualcomputing/viscomp2022) under Exercises/W3

To avoid merging issues with your solution from last week, every week you can pull the new exercise using the version_control.ipynb notebook.
(Or you can always clone the repository again)

Assignments:

1. Implement gradient thresholding edge detection

2. Implement Canny edge detection