Week 4 Tutorial

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Topics:

- Fourier transform principles
- Filtering in spatial and frequency domain
- Low Pass filter
- High Pass filter
- Band Pass filter
- Sampling

Fourier Transform

- Represents signal as a sum of periodic signals (e.g. sine)
- An image is a 2D function $I(x, y)$
- Fourier transform:

$$F(I)(u,v) = \iint_{R^2} I\,(x,y)\mathrm{e}^{-i2\pi(ux+vy)}dxdy$$

- Inverse Fourier transform:

$$I(x,y) = \iint_{R^2} F\,(I)(u,v)\mathrm{e}^{i2\pi(ux+vy)}dudv$$

$i^2 = -1$

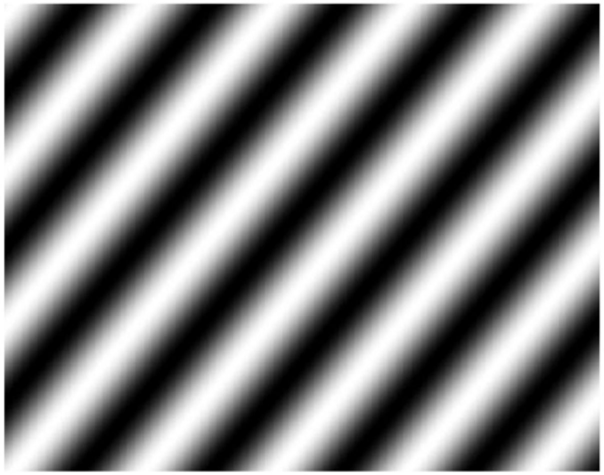
Fourier Transform on Images

- Intuition: the image I is decomposed into a weighted sum of 2D basis functions:

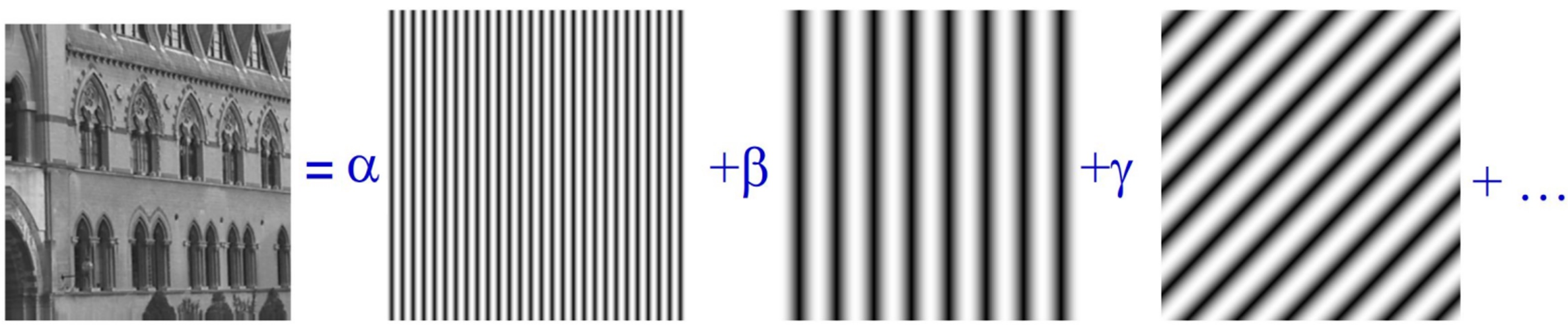
$$F(I)(u,v) = \iint_{R^2} I\,(x,y)\mathrm{e}^{-i2\pi(ux+vy)}dxdy$$

$$B_{u,v}(x,y) = \mathrm{e}^{-i2\pi(ux+vy)} = \cos\big(2\pi(ux+vy)\big) - i\sin\big(2\pi(ux+vy)\big)$$

- Vector (u,v)
- Magnitude ~ frequency
- Direction ~ orientation



Fourier Transform on Images



Credits: http://www.robots.ox.ac.uk/~az/lectures/ia/lect2.pdf

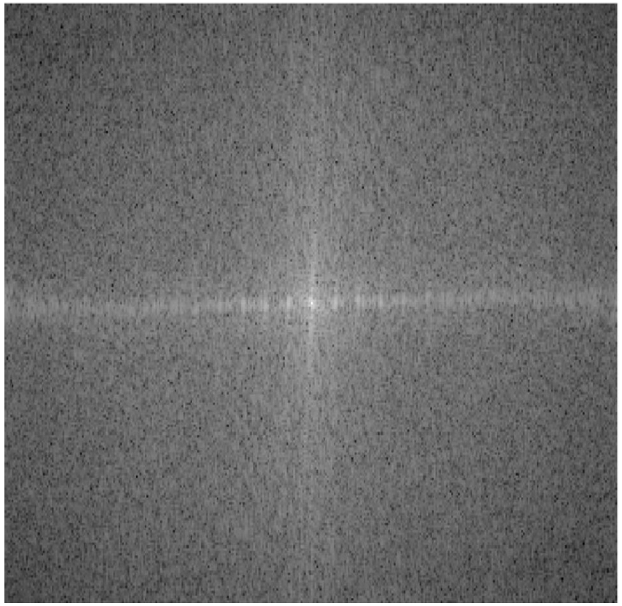
Fourier Transform and Convolution

Spatial Domain		Frequency Domain
$f * g$	$\longleftrightarrow$	$F \cdot G$
$f \cdot g$	$\longleftrightarrow$	$F * G$

- Convolution in **spatial** domain = multiplication in **frequency** domain and **vice-versa**.
- We can filter by applying Fourier Transform, multiplying and transforming back.

Frequencies in images

- Low image frequencies = slow gray level changes
- High image frequencies = fast changes in gray levels (e.g. edges and noise)

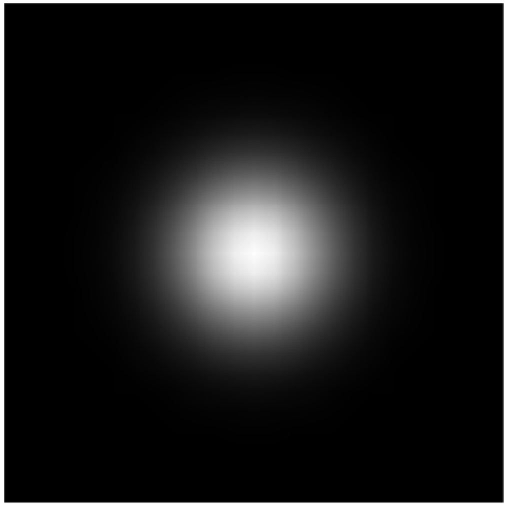


Task 1: Filtering

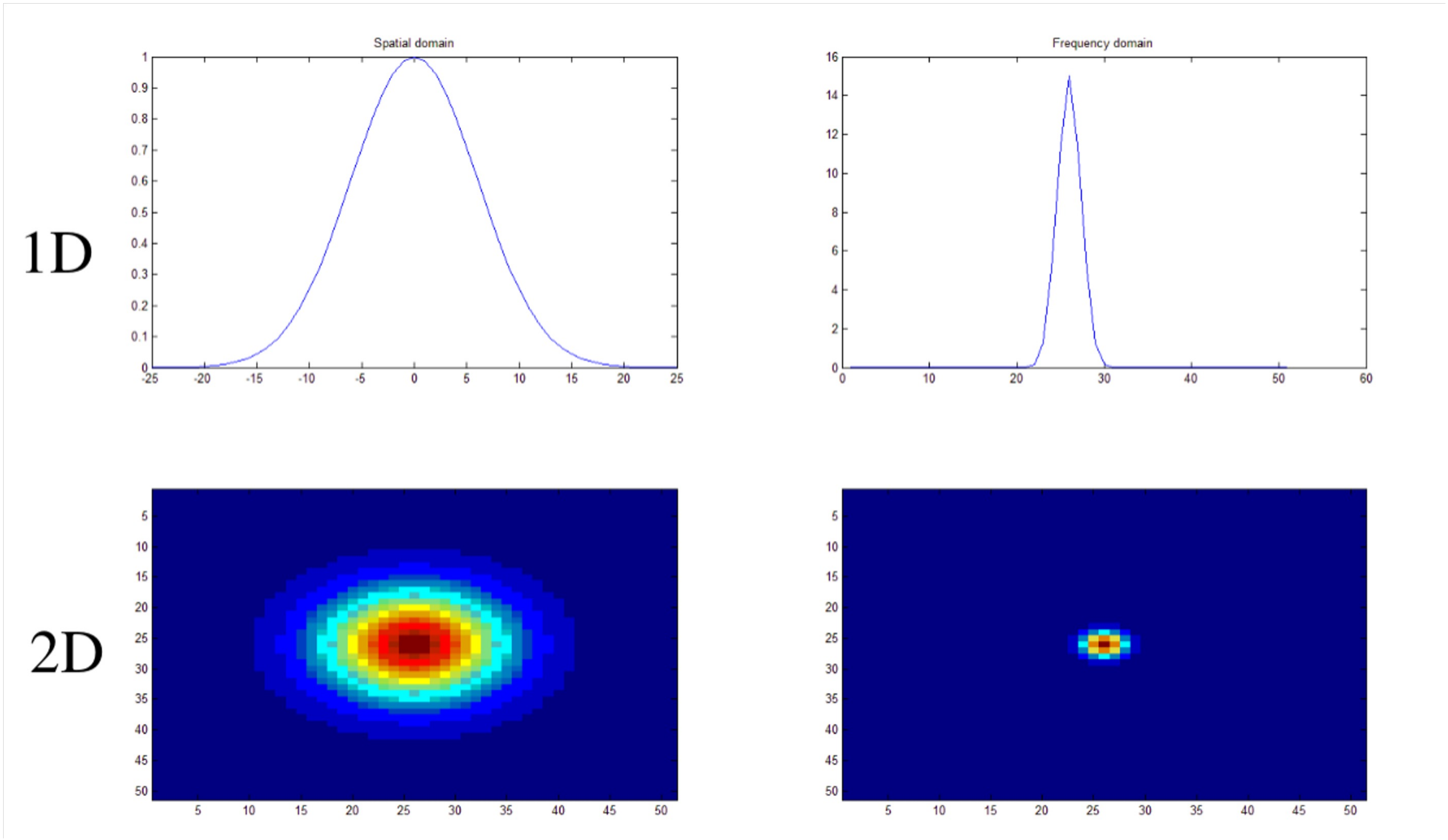
- Low pass filter
- High pass filter
- Band pass filter

Low pass filter

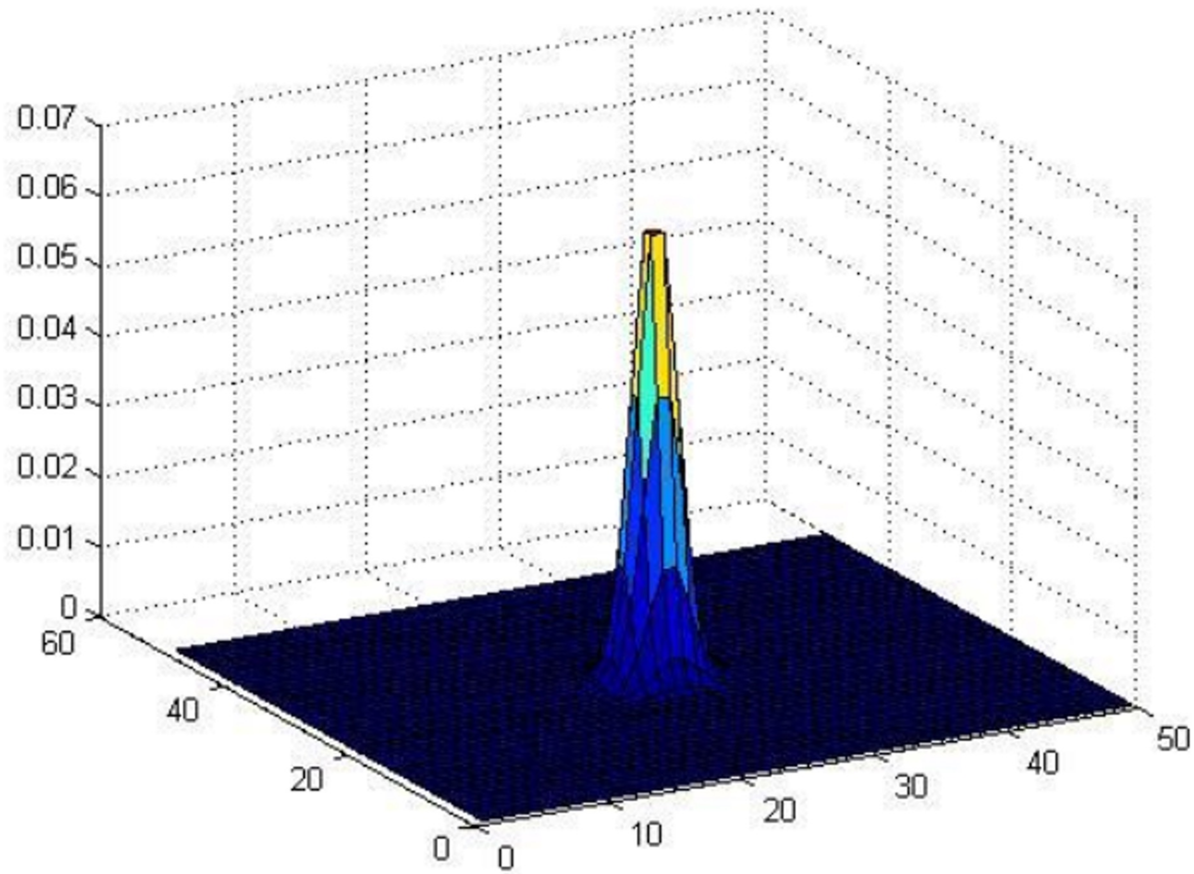
- Suppresses high frequencies, Retains low frequencies unchanged.
- Blocked high frequencies correspond to sharp intensity changes (fine-scale details, edges, noise)
- Result equivalent to smoothing.



Low pass filter: Gaussian

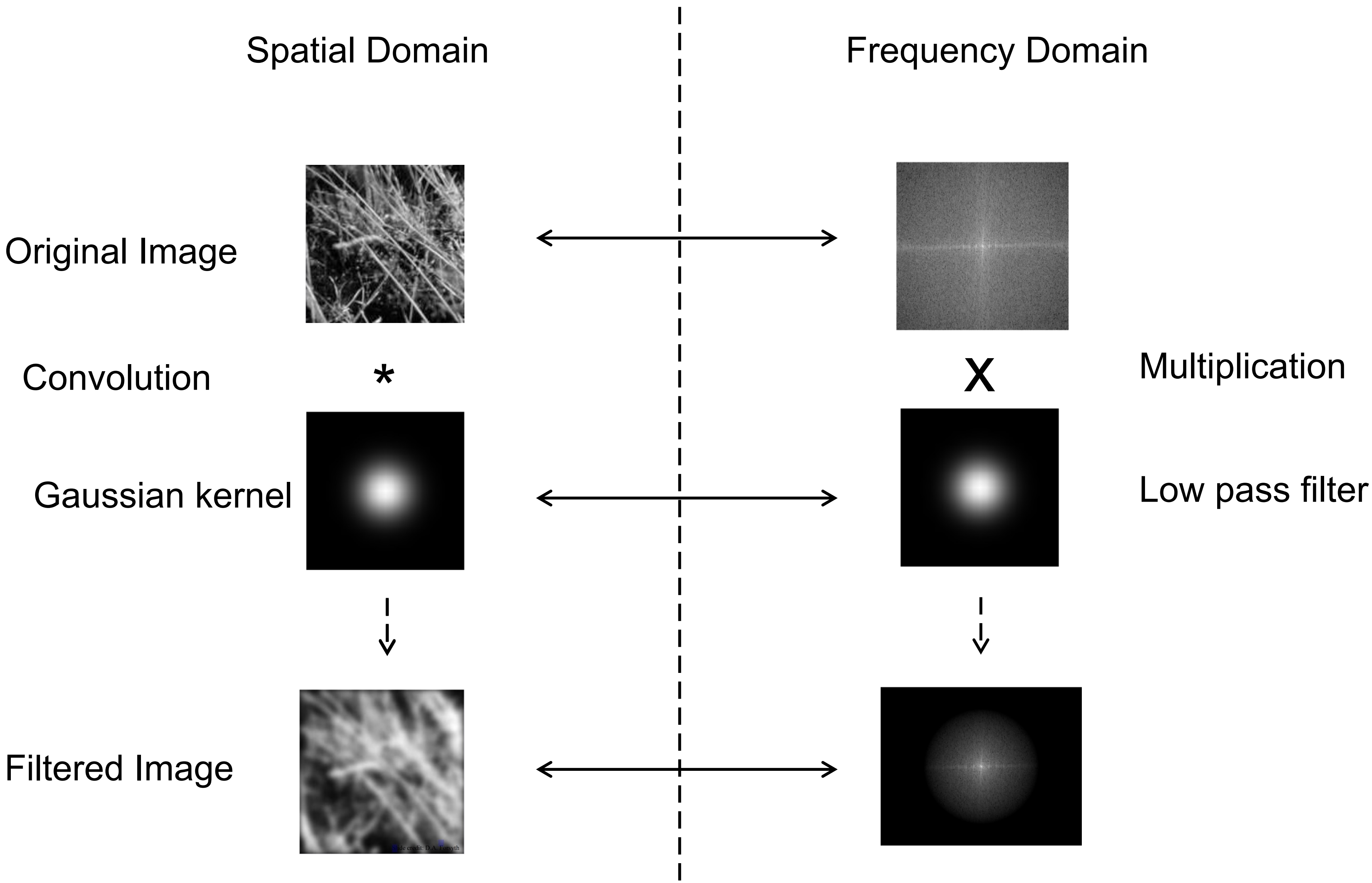


Low pass filter: Gaussian



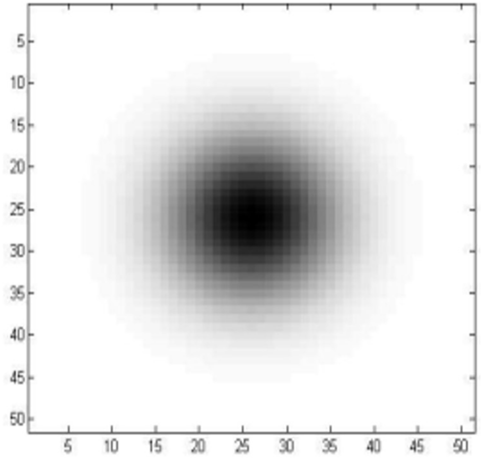
Gaussian kernel of size (50, 50) and standard deviation 2.5

Low pass filter



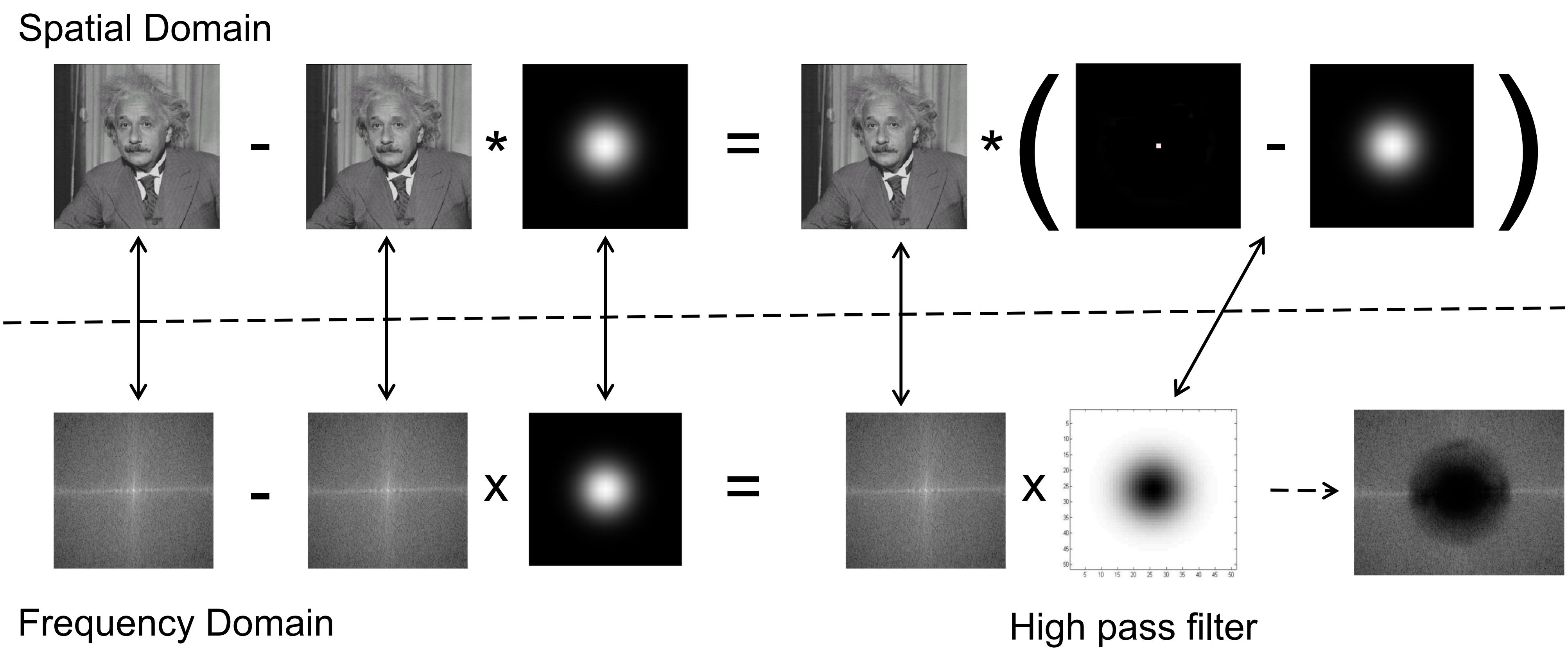
High pass filter

- Suppresses low frequencies, Retains high frequencies unchanged.
- Edges are enhanced
- Suppressed low frequencies correspond to areas of constant gray level
- Result equivalent to difference between original image and image filtered by Gaussian.
- Frequency complement of the low pass filter.



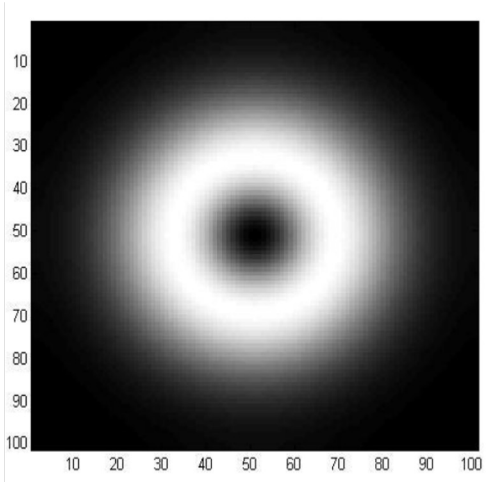
High pass filter

- Result equivalent to difference between original image and image filtered by Gaussian.
- Frequency complement of the low pass filter.



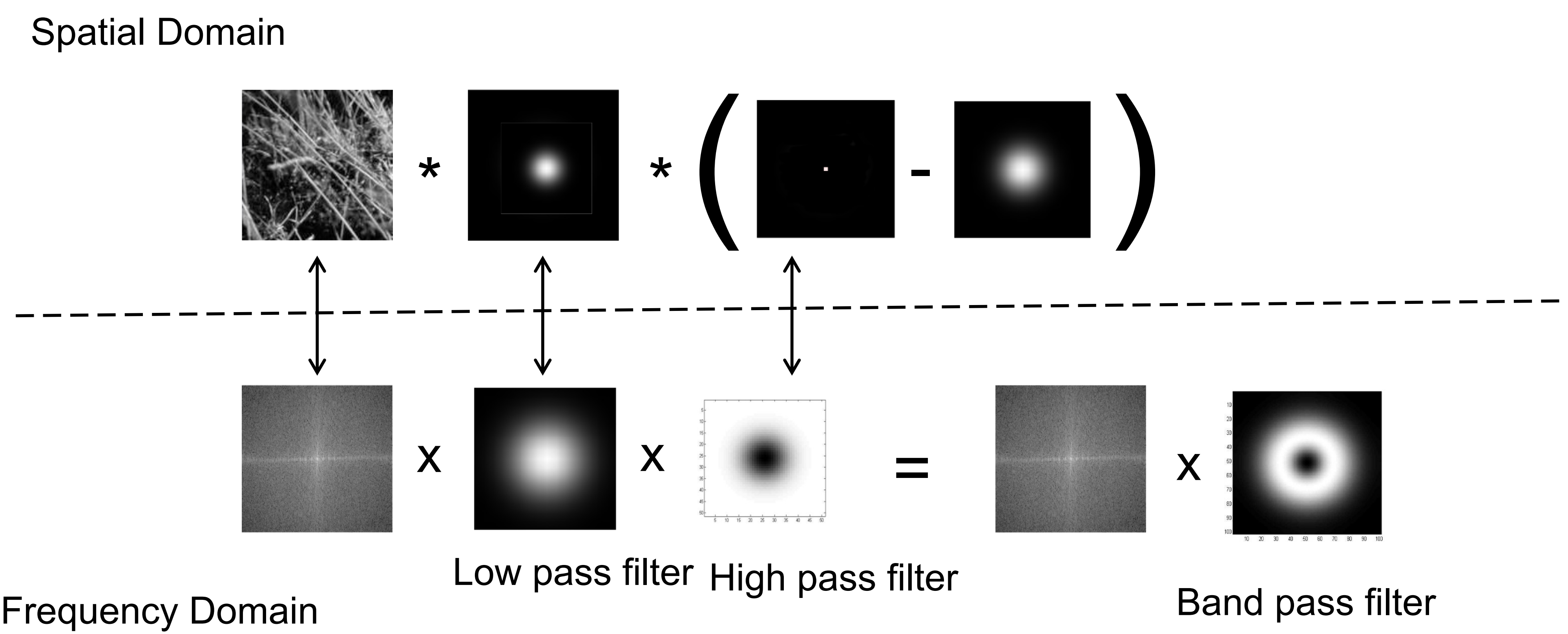
Band pass filter

- Suppresses both the low frequencies (< D0) and the high frequencies (> D1).
- Retains the middle range band of frequencies.
- May be used to enhance edges (suppressing high frequencies) while reducing the noise (suppressing low frequencies)
- Result equivalent to successive filtering by low pass filter and high pass filter



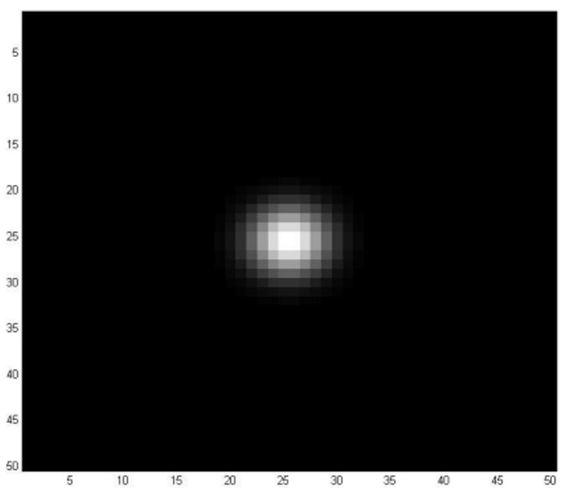
Band pass filter

- Result equivalent to successive filtering by low pass filter and high pass filter

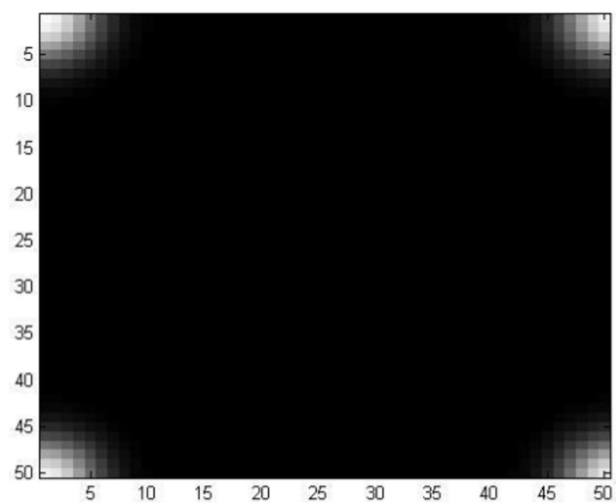


Fourier Transform in Python

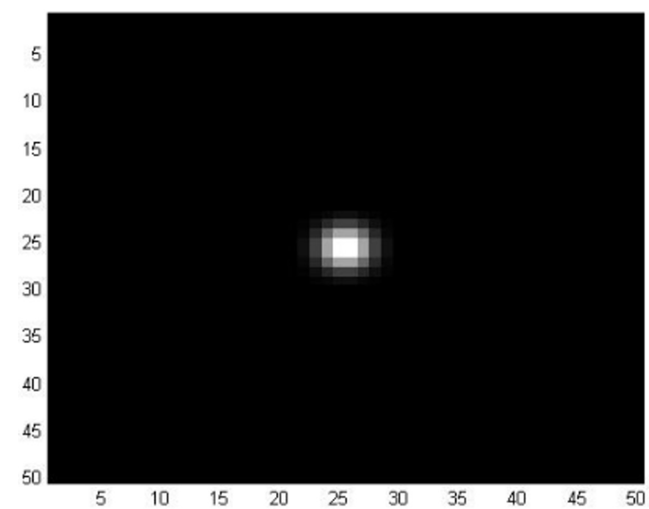
- Command fft2: runs Fast Fourier Transform on image
- In Python the low frequencies are displayed at the corners
- `scipy.fftpack.fftshift` brings the origin to the center of the image




```
filter = gaussian_filter((50,50),1.5)
imshow(filter, cmap='gray')
```




```
h = fft2(filter)
imshow(numpy.abs(h),cmap='gray')
```




```
h = fftshift(fft2(filter))
imshow(numpy.abs(h),cmap='gray')
```

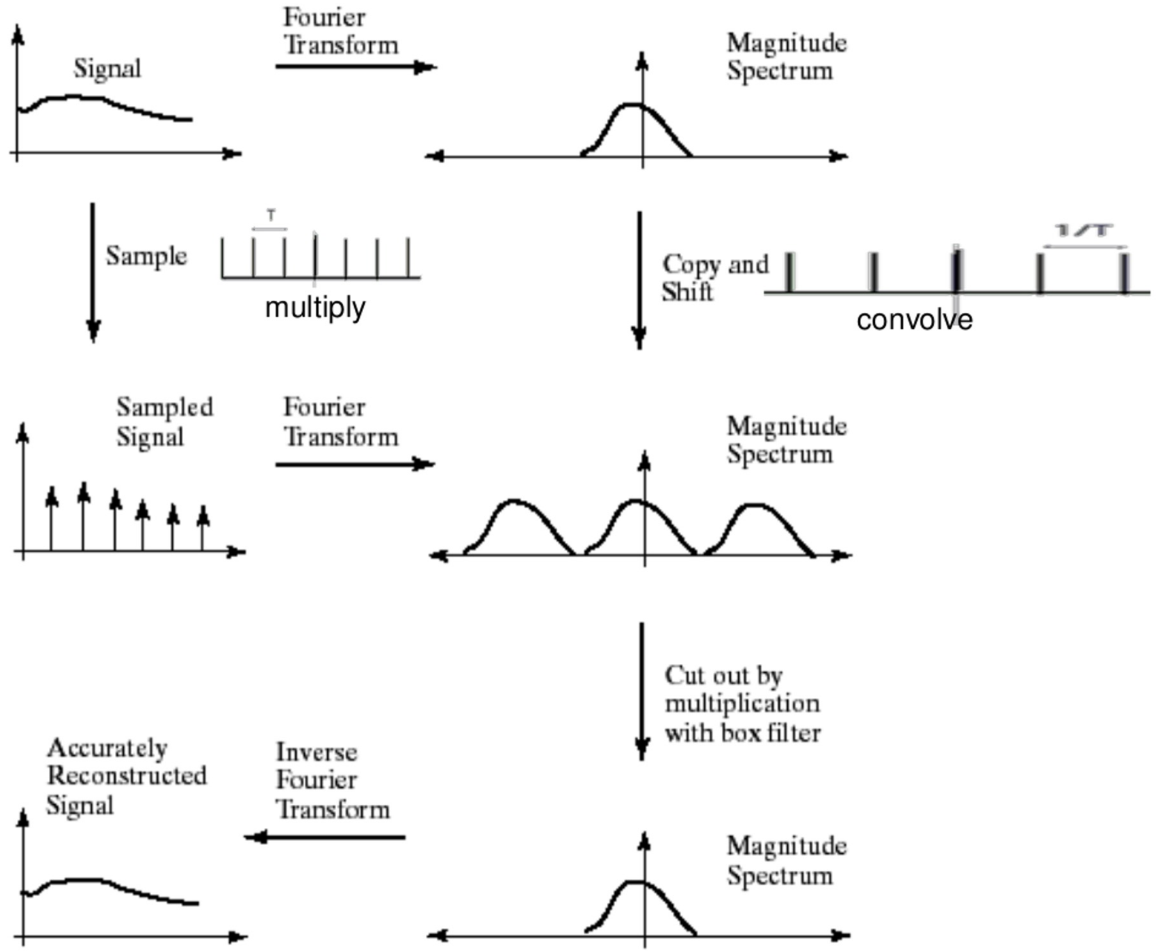
Task 2: Sampling

- Sampling:
  - Discretization of the image by measuring values on a regular grid

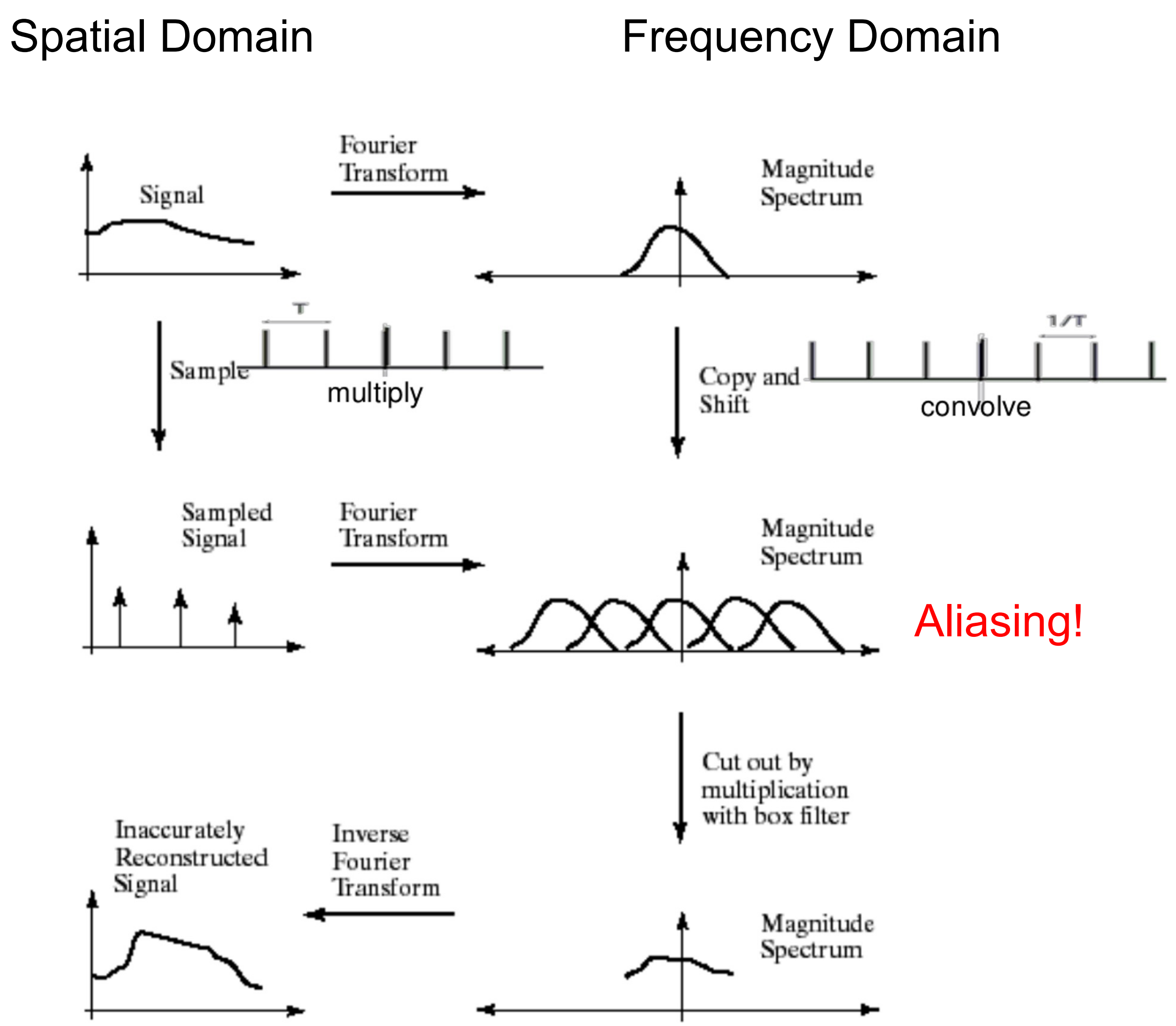
- Nyquist–Shannon sampling theorem
  - Sampling frequency has to be at least 2x the highest frequency in image
  - If not fulfilled, **aliasing** appears.

Sampling scheme

Spatial Domain   Frequency Domain



Aliasing


Spatial Domain
Frequency Domain
Fourier Transform
Signal
Magnitude Spectrum
T
Sample
multiply
1/T
Copy and Shift
convolve
Sampled Signal
Fourier Transform
Magnitude Spectrum
Aliasing!
Cut out by multiplication with box filter
Inaccurately Reconstructed Signal
Inverse Fourier Transform
Magnitude Spectrum

Avoiding aliasing

- To avoid aliasing, the Nyquist-Shannon theorem must hold.
  - Sampling frequency $\geq$ 2x the highest frequency in image

- **Solution**: Reduce the maximal frequency in data
  - Use the low-pass filter

Bonus task

- Blurred image of street signs
- Use one of the presented methods to make the text readable