

# Visual Computing Exercise 7: Light & Colors

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# Exercise 7 - Light and Colors

- Theoretical – no programming
- Good for the exam
- Should be solved individually
- Goals
  - Definitions
  - Color Spaces
  - CIE chromaticity diagram

# DEFINITIONS

# Measuring Light

- Luminous Flux  $F$  [lumen]

Perceived power of light

$$F = \text{const} \cdot \int_{380nm}^{780nm} P(\lambda) V(\lambda) d\lambda$$

$P(\lambda)$ : Relative Spectral Power Density

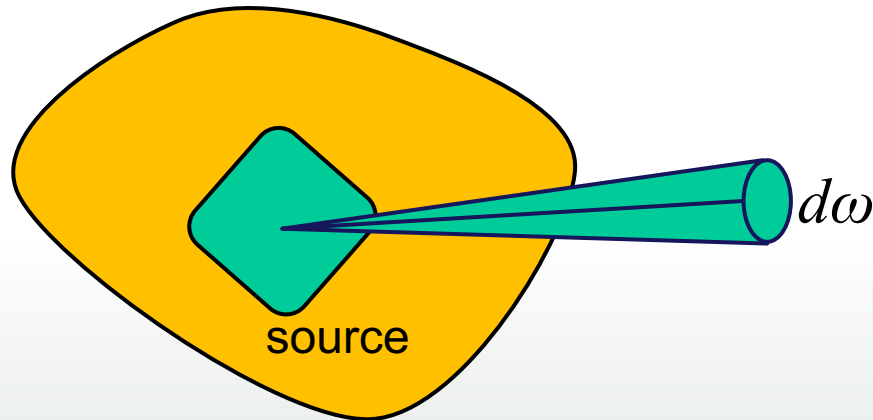
$V(\lambda)$ : Relative Spectral Sensitivity

$$\text{const} : 683 \frac{\text{lm}}{\text{W}}$$

# Measuring Light

- Luminous Intensity  $I$  [candela]

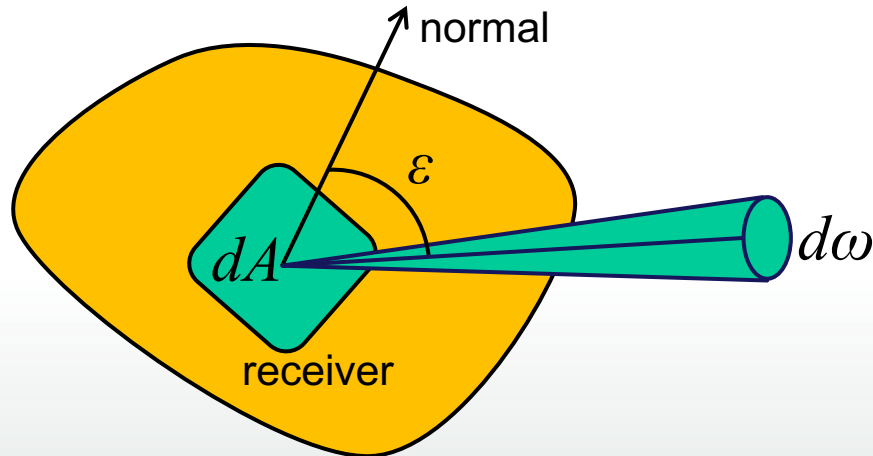
$$I = \frac{dF}{d\omega} \quad F = \int I d\omega$$



# Measuring Light

- Luminance  $Y$  [candela/m<sup>2</sup>]

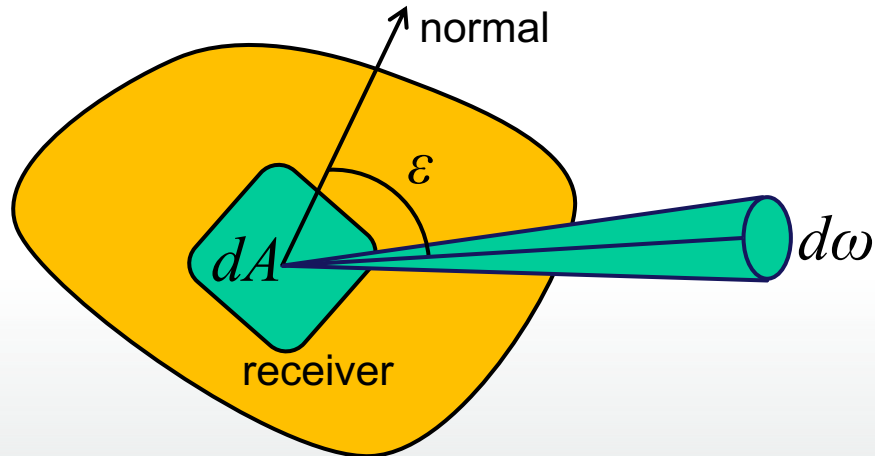
$$Y = \frac{d^2F}{dA \cos \varepsilon d\omega} \quad I = \int Y dA \quad F = \iint Y dA d\omega$$



# Measuring Light

- Illumination  $B$  [lux]

$$B = \frac{dF}{dA}$$



## Luminous Flux $F$ [lumen]

$$F = c \cdot \int_{380nm}^{780nm} P(\lambda) V(\lambda) d\lambda$$

## Luminous Intensity $I$ [candela]

$$I = \frac{dF}{d\omega}$$

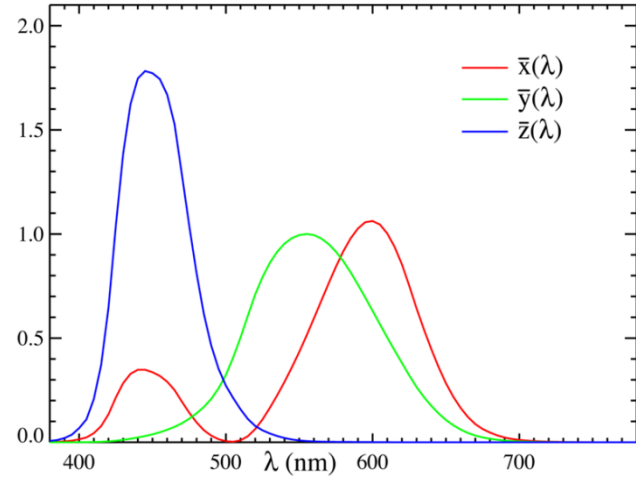
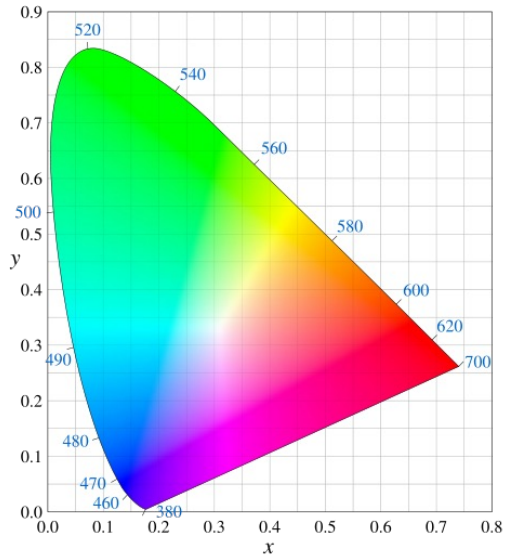
## Luminance $Y$ [candela / m<sup>2</sup>]

$$Y = \frac{d^2F}{dA \cos \varepsilon d\omega}$$

## Illumination $B$ [lux]

$$B = \frac{dF}{dA}$$





# THE CIE CHART

# Overview

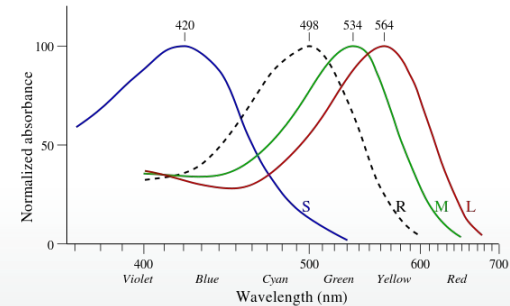
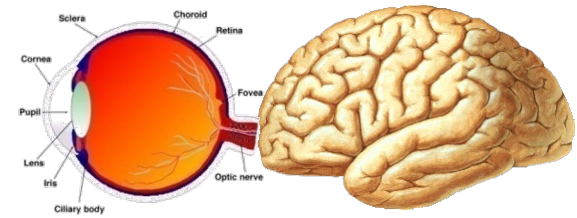
- What is color?

Real World



vs.

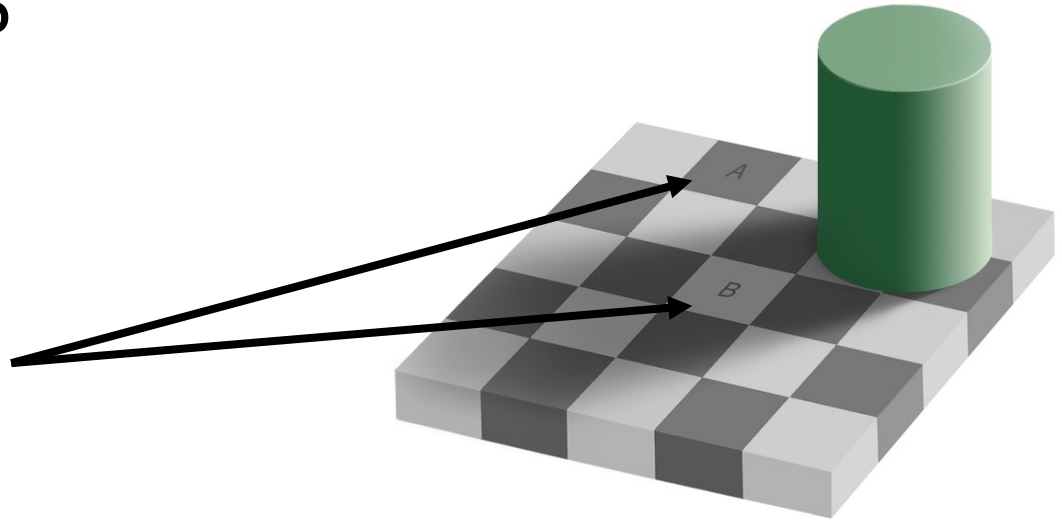
Perception



# Overview

- What is color?

Have the  
same color



# Overview

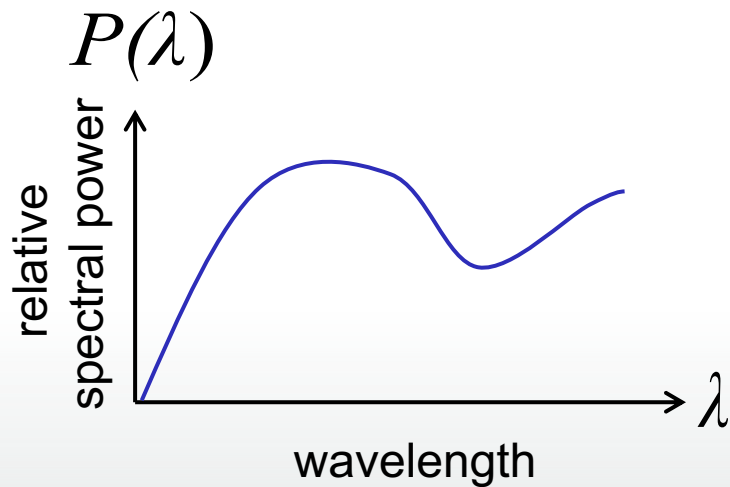
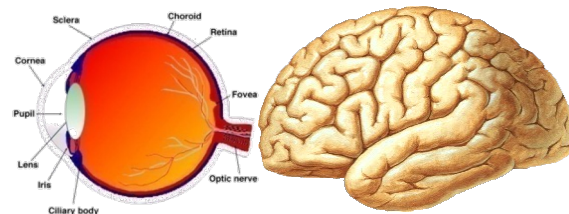
- Light can be described by frequencies

$$P(\lambda)$$

- Light containing certain frequencies is perceived as colors

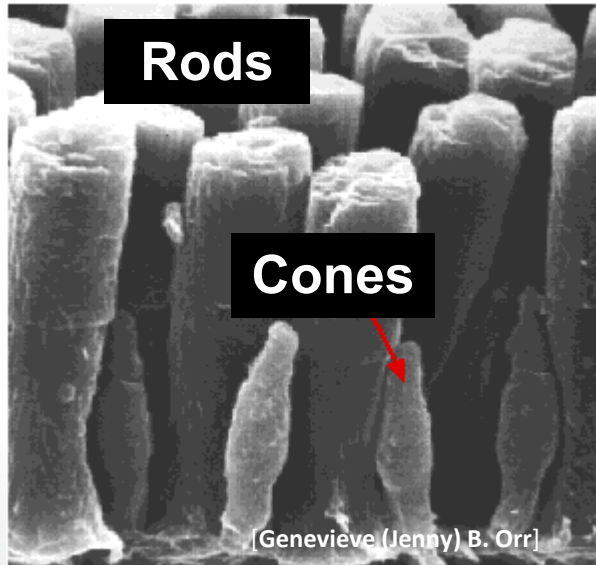
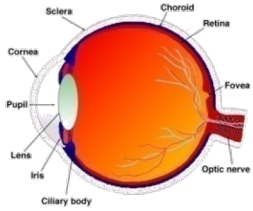
# Overview

- What is the mapping from frequencies to colors?

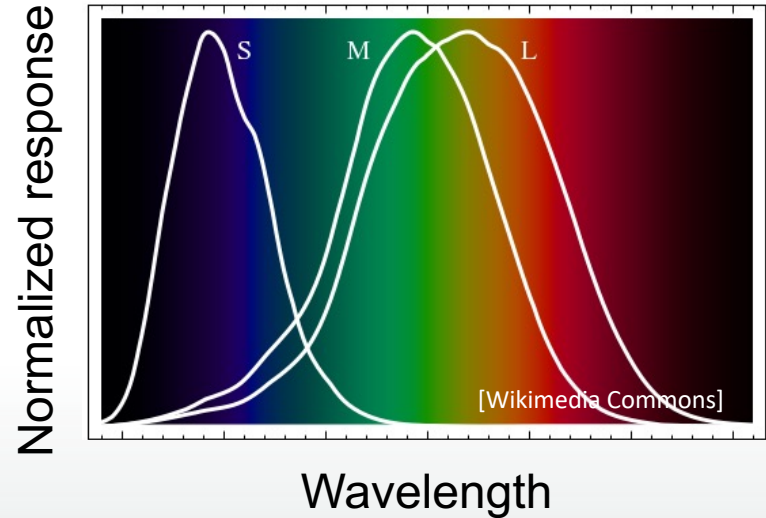


# Overview

- The human eye has photosensitive cells

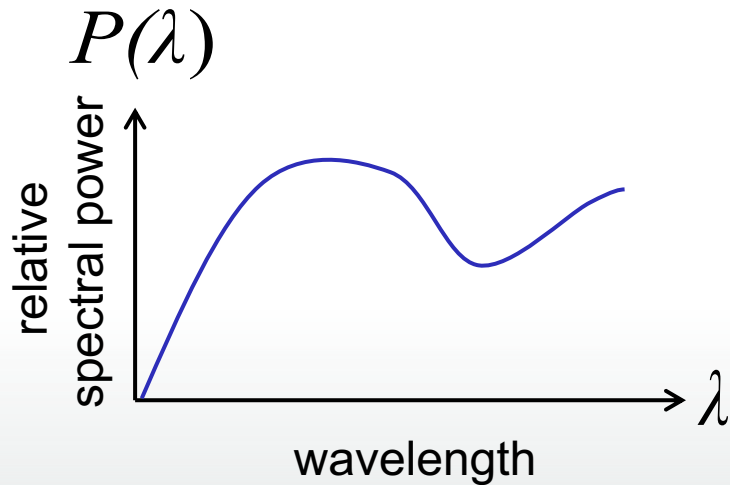


Cones: S,L,M Type



# Overview

- Only three cone cell types
- Humans project the light signal into  $R^3$



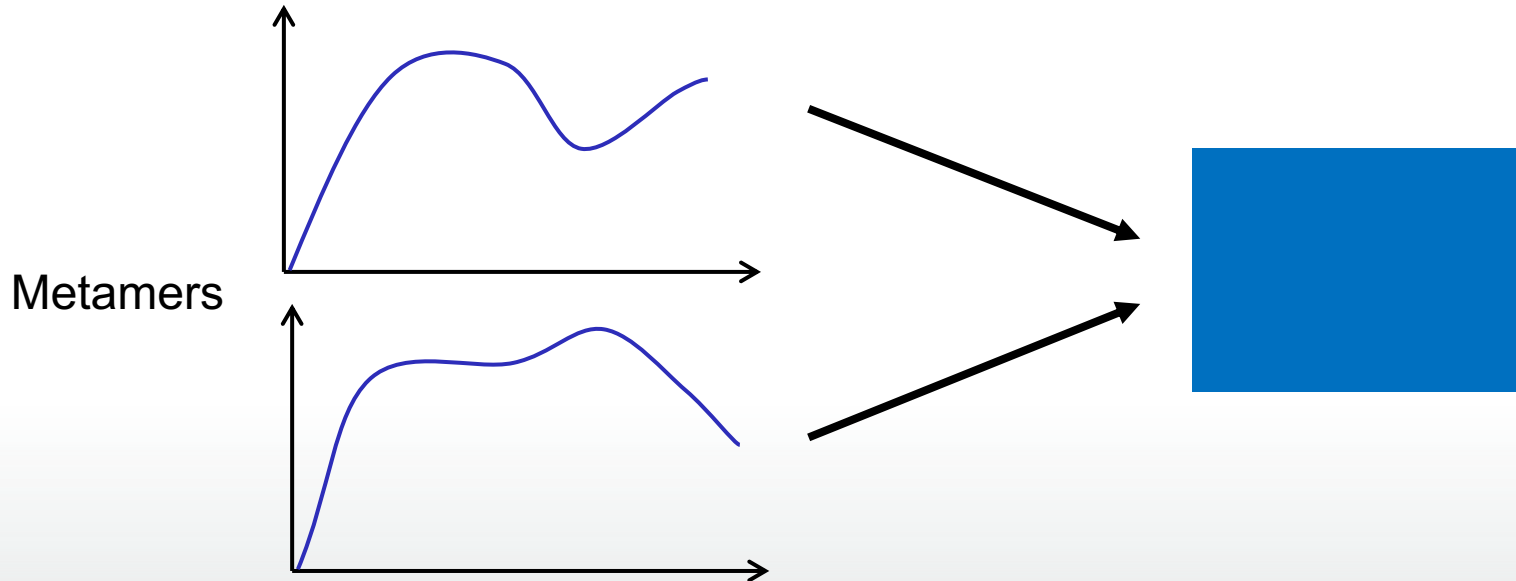
$$\int_{380nm}^{780nm} P(\lambda) S(\lambda) d\lambda$$

$$\int_{380nm}^{780nm} P(\lambda) L(\lambda) d\lambda$$

$$\int_{380nm}^{780nm} P(\lambda) M(\lambda) d\lambda$$

# Overview

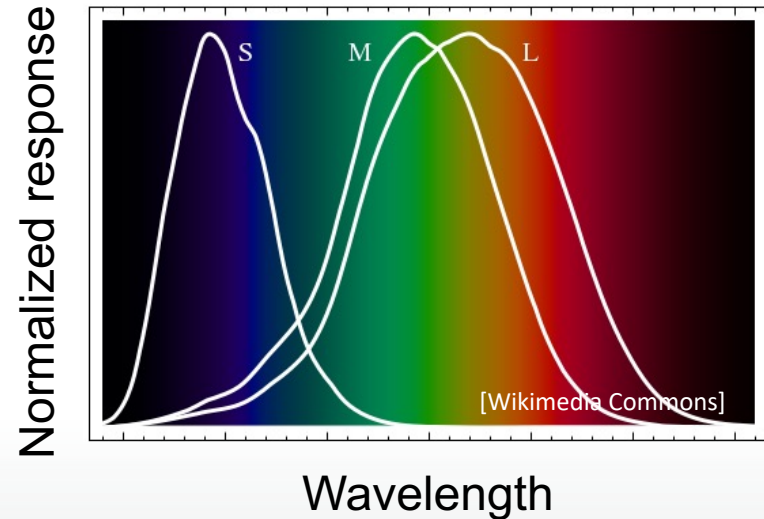
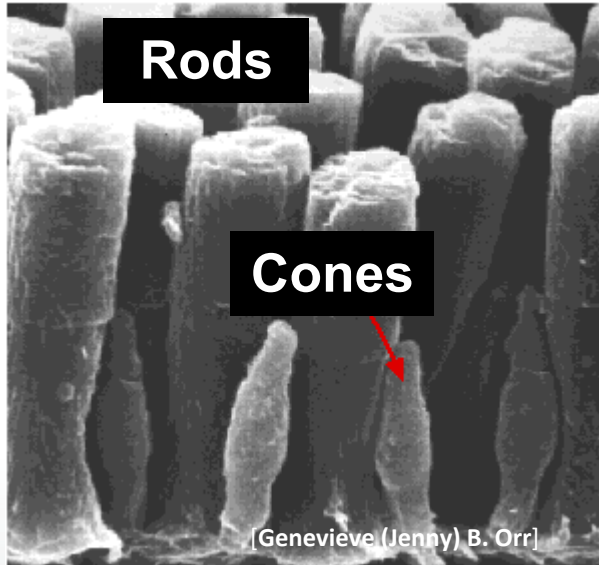
- Light with different spectrum can map to the same color: metamers





# Overview

- How to determine these responses?



# Experiment (CIE, 1931)

Primaries (single frequency light)

$$r(\lambda) = \delta(\lambda - 700)$$



700nm

$$g(\lambda) = \delta(\lambda - 546)$$



546nm

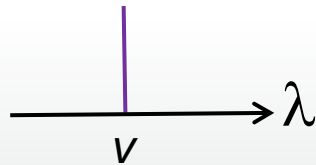
$$b(\lambda) = \delta(\lambda - 435)$$



435nm

Reference

$$L(\lambda) = \delta(\lambda - v)$$

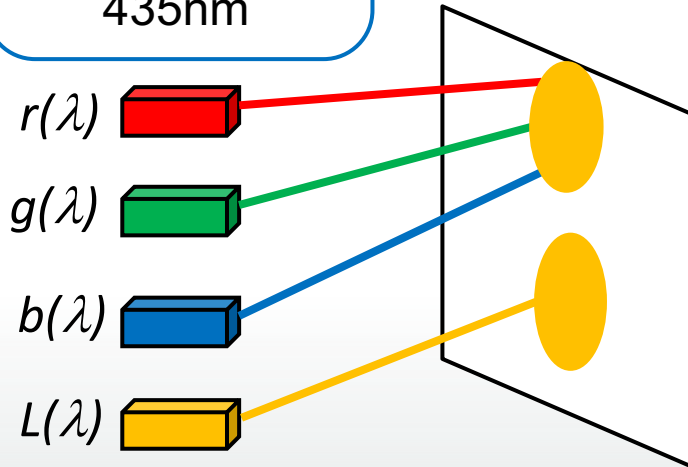


$r(\lambda)$  

$g(\lambda)$  

$b(\lambda)$  

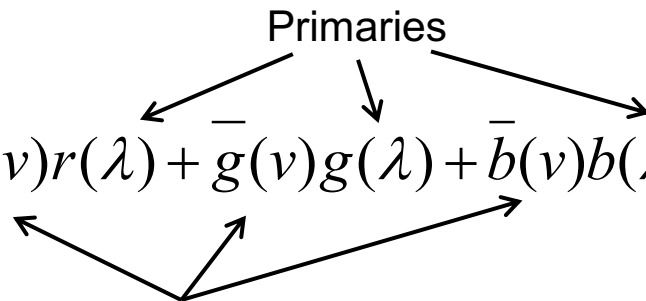
$L(\lambda)$  



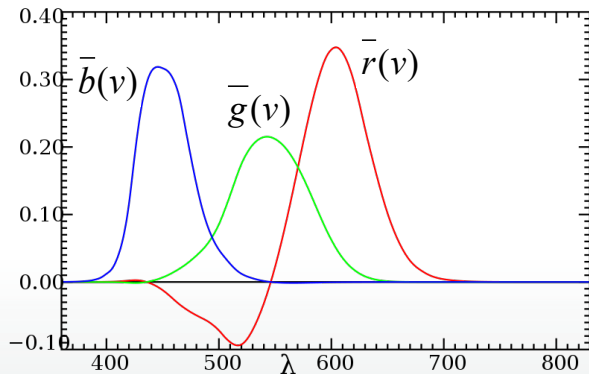
# Experiment (CIE, 1931)

Perceptual equivalence

Reference  $\longrightarrow \delta(\lambda - \nu) \hat{=} \bar{r}(\nu)r(\lambda) + \bar{g}(\nu)g(\lambda) + \bar{b}(\nu)b(\lambda)$

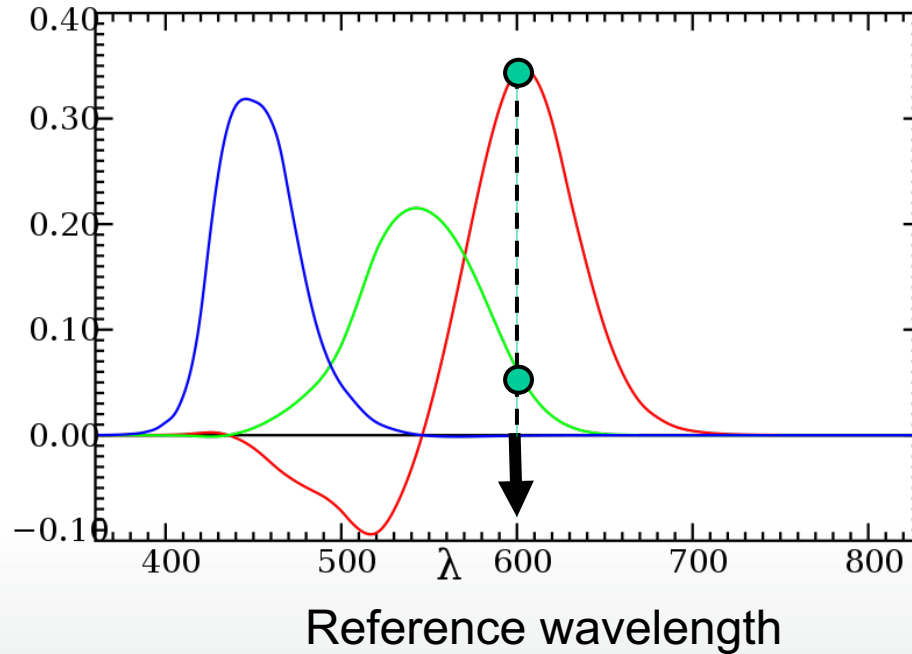


Color matching functions



# Experiment (CIE, 1931)

Experiment gives: how much of each primary is required for the given reference



# Experiment (CIE, 1931)

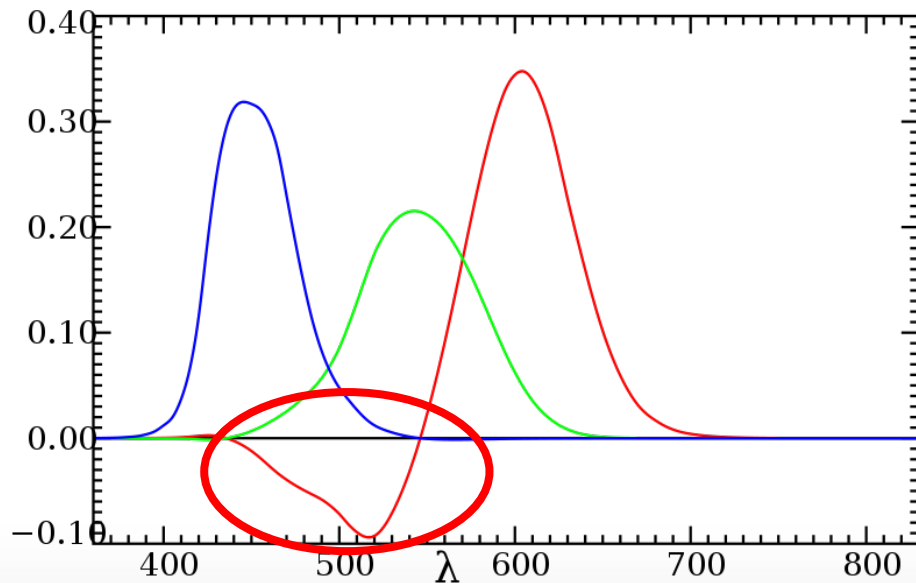
For arbitrary spectral power density function  $P(\lambda)$

$$P(\lambda) = \int P(\nu)\delta(\lambda - \nu)d\nu \hat{=} \int P(\nu)(\bar{r}(\nu)r(\lambda) + \bar{g}(\nu)g(\lambda) + \bar{b}(\nu)b(\lambda))d\nu$$
$$= \underbrace{\left(\int P(\nu)\bar{r}(\nu)d\nu\right)}_R \cdot r(\lambda) + \underbrace{\left(\int P(\nu)\bar{g}(\nu)d\nu\right)}_G \cdot g(\lambda) + \underbrace{\left(\int P(\nu)\bar{b}(\nu)d\nu\right)}_B \cdot b(\lambda)$$

Hence, any color can be generated

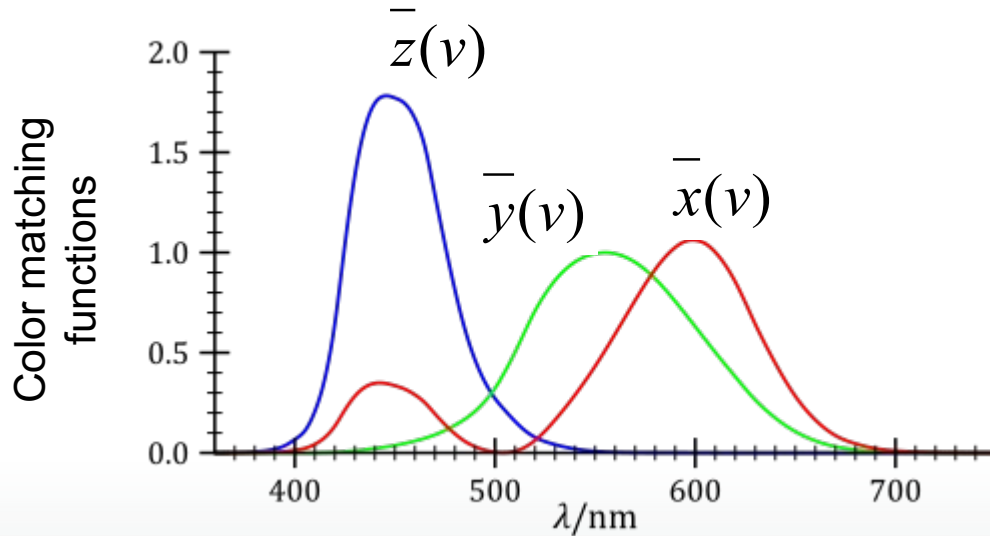
# Experiment (CIE, 1931)

Problem: negative color matching function. Cannot emit negative light!



# Experiment (CIE, 1931)

Solution: new primaries and color matching functions  
But imaginary primaries



# Experiment (CIE, 1931)

Solution: new primaries and color matching functions  
They define the CIE XYZ space

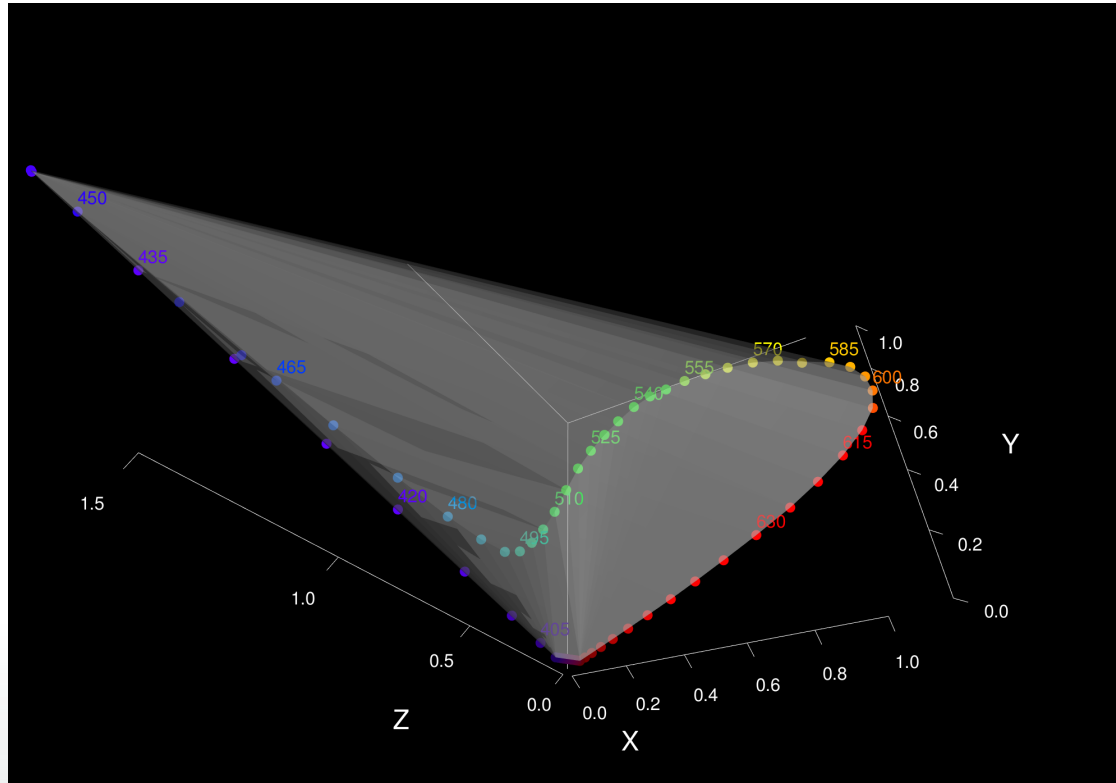
$$X = \int_{380nm}^{780nm} P(\lambda) \bar{x}(\lambda) d\lambda$$

$$Y = \int_{380nm}^{780nm} P(\lambda) \bar{y}(\lambda) d\lambda$$

$$Z = \int_{380nm}^{780nm} P(\lambda) \bar{z}(\lambda) d\lambda$$



# CIE XYZ in 3D



[source: wiki commons](#)

# The CIE Chromacity Diagram

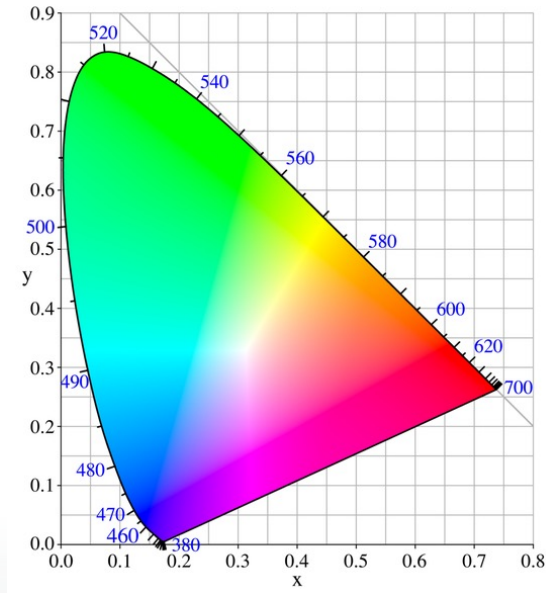
- X, Y, Z contain luminance
- Normalize to get chromacity: x, y, Y

$$x = \frac{X}{X + Y + Z} \quad y = \frac{Y}{X + Y + Z} \quad Y$$

- Plot in 2D: The CIE Chromacity Diagram

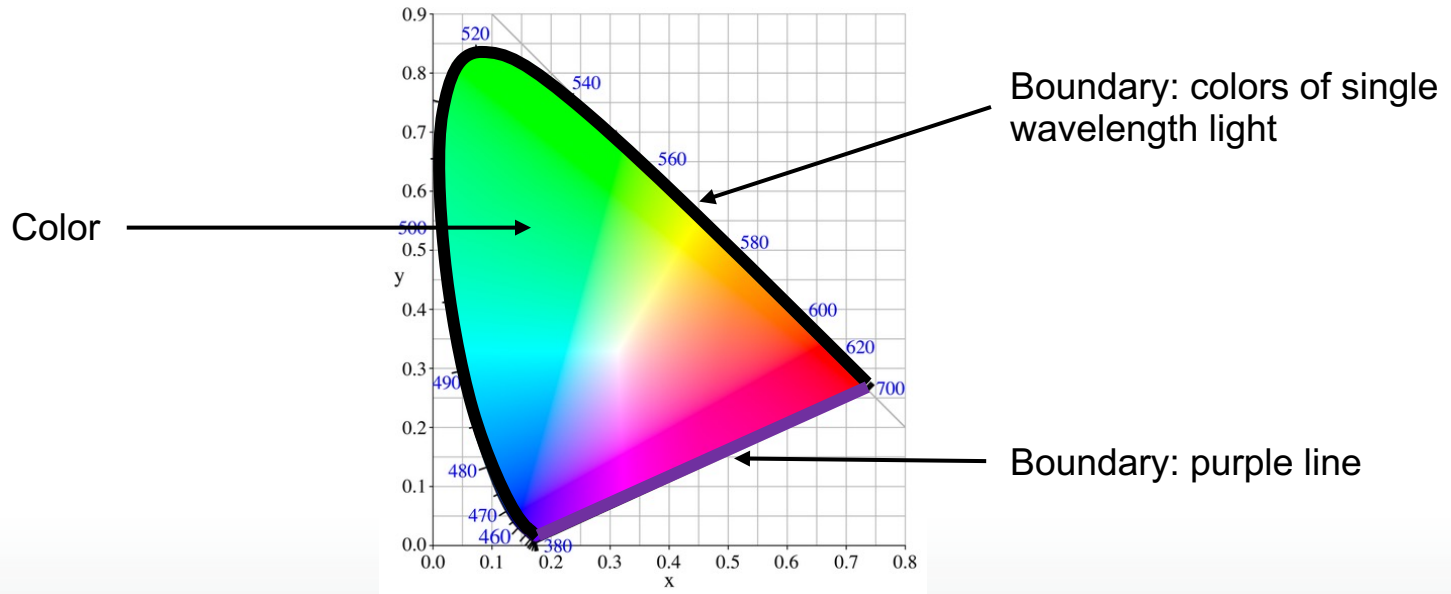
# The CIE Chromacity Diagram

- Plot in 2D: the CIE Chromacity Diagram



# The CIE Chromacity Diagram

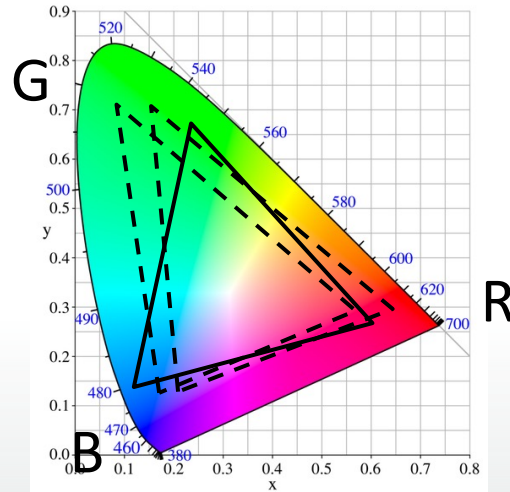
- Plot in 2D: the CIE Chromacity Diagram





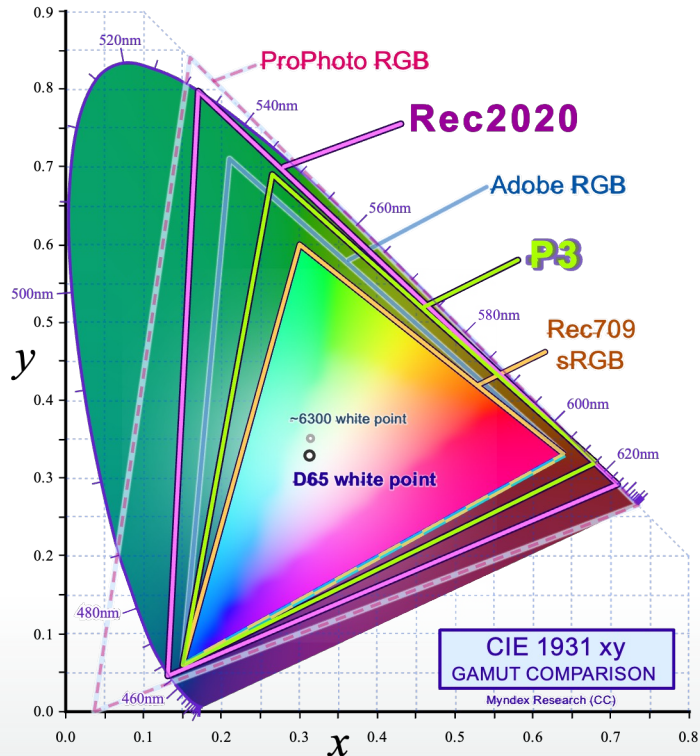
# CIE XYZ and RGB

- While CIE XYZ is a canonical color space, images/devices rarely work directly with XYZ.
- RGB primaries dominate the industry, this is because we can produce RGB light sources (LEDs, phosphorus for CRT monitors, filters, etc)

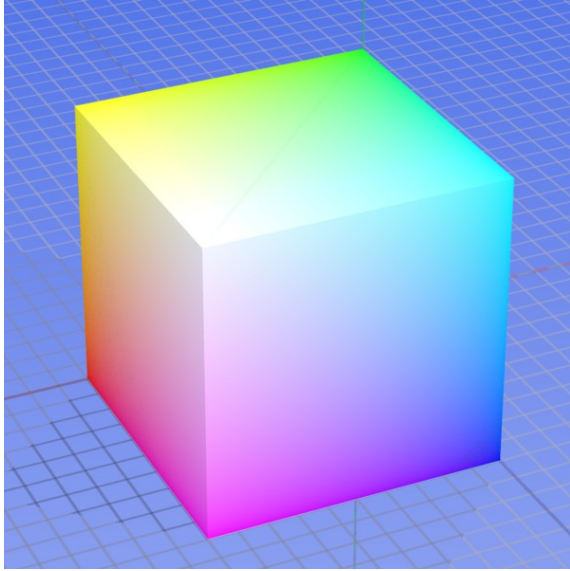


# Standard RGB (sRGB) – Rec. 709

- In 1996, Microsoft and HP defined a set of “standard” RGB primaries.
- This was considered an RGB space achievable by most devices at the time.



[source: wiki commons](#)

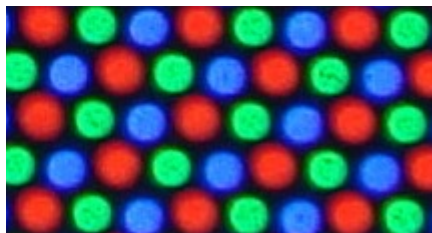


A BALANCED COLOR SPHERE

# OTHER COLOR SPACES



# RGB and XYZ



	<b>R</b>	<b>G</b>	<b>B</b>
$x$	$x_R=0.64$	$x_G=0.30$	$x_B=0.15$
$y$	$y_R=0.33$	$y_G=0.60$	$y_B=0.06$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} x_R C_R & x_G C_G & x_B C_B \\ y_R C_R & y_G C_G & y_B C_B \\ (1 - x_R - y_R) C_R & (1 - x_G - y_G) C_G & (1 - x_B - y_B) C_B \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

# White point calibration

- Find the parameters  $C_R$ ,  $C_G$ ,  $C_B$

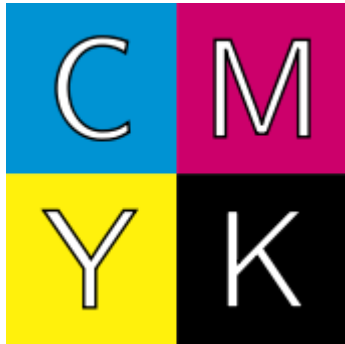
White point specification

RGB White point =  $(1, 1, 1)^T$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} x_R C_R & x_G C_G & x_B C_B \\ y_R C_R & y_G C_G & y_B C_B \\ (1 - x_R - y_R) C_R & (1 - x_G - y_G) C_G & (1 - x_B - y_B) C_B \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

# CMY(K) – Subtractive for printers

- Cyan, Magenta and Yellow as basis vectors
- Key as additional color



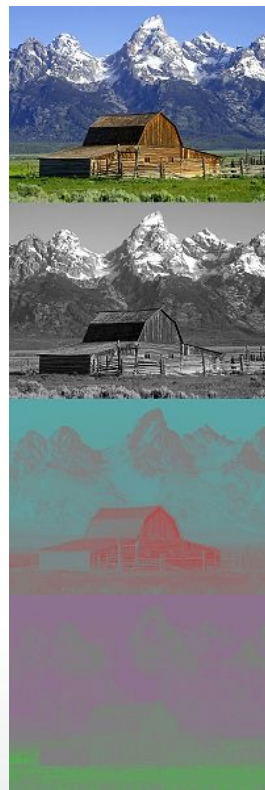
$$\begin{pmatrix} C \\ M \\ Y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

resp.

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} C \\ M \\ Y \end{pmatrix}$$

# YIQ – Perceptual

- NTSC US-color TV standard
- Y as luma information  
(used by b/w televisions)
- I for in-phase (orange-blue)  
(natural skin color)
- Q for quadrature (purple-green)
- Bandwidth partitioning  
(2.4MHz, 1.5MHz, 0.6MHz)
- Check lecture slides for the linear transformation from RGB



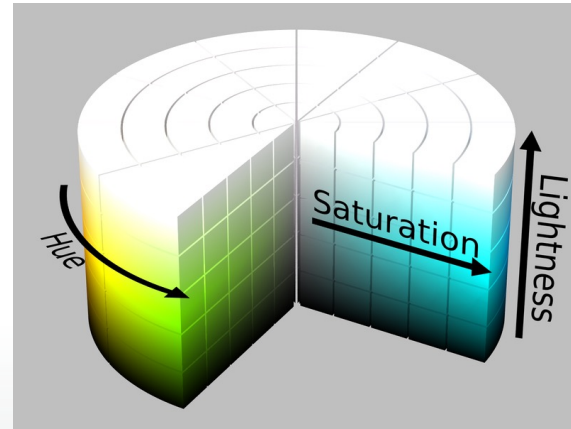
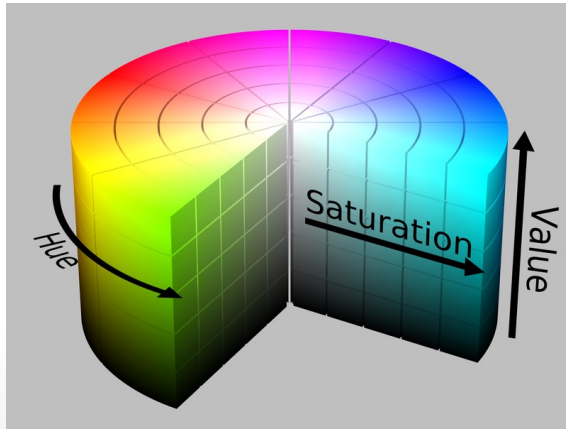
Y

I

Q

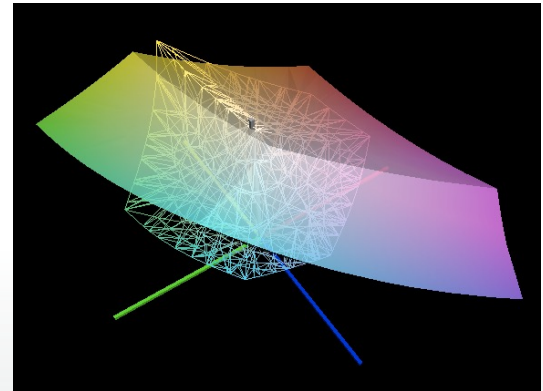
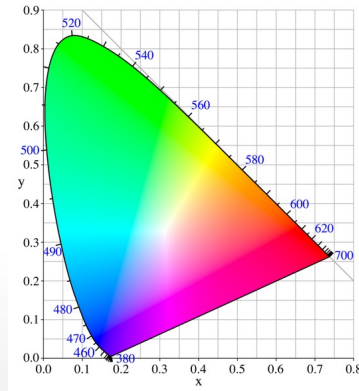
# HSL/HSV – Intuitive picking

- Perceptually motivated
- Hue, Saturation, Lightness (Value)
- Simple conversion from RGB (check slides)



# L\*a\*b / Luv – Perceptual distance

- CIE does not provide perceptually correct distances
- Determine ellipses of approximately the same color
- Map them to circles
- L\*a\*b and Luv are nonlinear warps of the CIE chart



# Visual Computing Exercise 7: Light & Colors

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