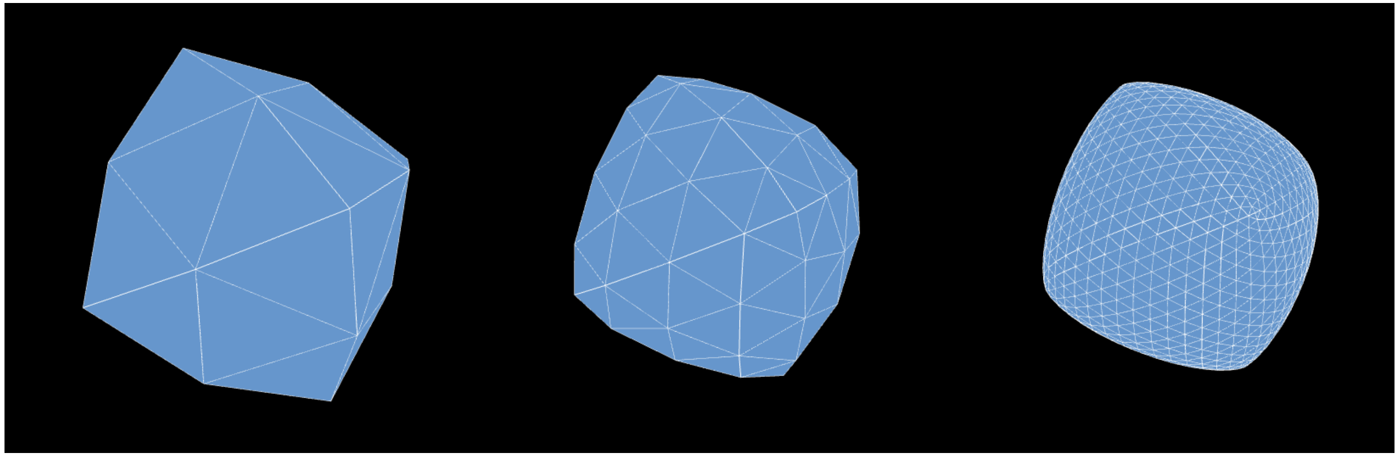
Visual Computing
Exercise 11: Curves & Surfaces



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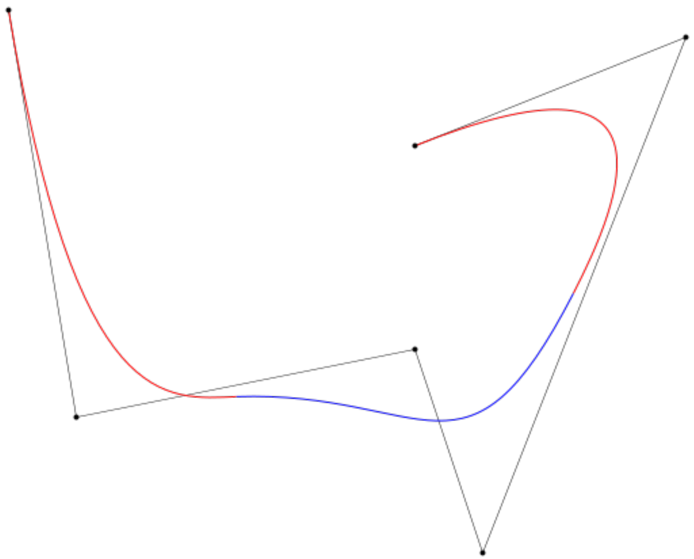
Recap

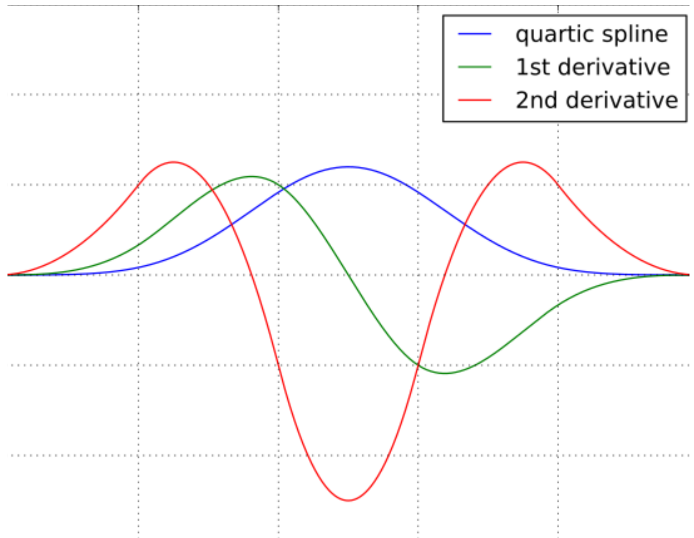
- Parameterization of curves
- Bezier Curves
- B-Spline Curves
- Subdivison Surfaces
  - Loop Subdivision

2D & 3D Curves

Curve $\boldsymbol{x}(u)$ as a map of the 1D parameter space u into 2D or 3D

$$\boldsymbol{x}(u) = (x(u), y(u), z(u))^T$$





source: https://en.wikipedia.org/wiki/File:B-spline_curve.svg
source: https://en.wikipedia.org/wiki/File:Cardinal_quartic_B-spline.svg

B-Spline Curves

- Disadvantages of Bézier curves:
  - Global support of the basis functions
  - Insertion of new control points comes along with degree elevation
  - C^r-continuity between individual segments of a Bézier curve

⇒ ***B-Spline bases help to overcome these problems***
(Local support, continuity control, arbitrary knot vector)

B-Spline Functions

- From the recurrence formula we obtain:

$$N_i^1(u) = \begin{cases} \dfrac{u-u_i}{u_{i+1}-u_i}, & u \in [u_i, u_{i+1}] \\ \dfrac{u_{i+2}-u}{u_{i+2}-u_{i+1}}, & u \in [u_{i+1}, u_{i+2}] \end{cases}$$

$$N_i^2(u) = \frac{u-u_i}{u_{i+2}-u_i} N_i^1(u) + \frac{u_{i+3}-u}{u_{i+3}-u_{i+1}} N_{i+1}^1(u)$$

$$= \begin{cases} \dfrac{u-u_i}{u_{i+2}-u_i} \cdot \dfrac{u-u_i}{u_{i+1}-u_i} & i \in [u_i, u_{i+1}] \\ \dfrac{u-u_i}{u_{i+2}-u_i} \cdot \dfrac{u_{i+2}-u}{u_{i+2}-u_{i+1}} + \dfrac{u_{i+3}-u}{u_{i+3}-u_{i+1}} \cdot \dfrac{u-u_{i+1}}{u_{i+2}-u_{i+1}}, & i \in [u_{i+1}, u_{i+2}] \\ \dfrac{u_{i+3}-u}{u_{i+3}-u_{i+1}} \cdot \dfrac{u_{i+3}-u}{u_{i+3}-u_{i+2}} & i \in [u_{i+2}, u_{i+3}] \end{cases}$$

B-Spline Functions

- Recurrence relation:

$$N_i^n(u) = (u - u_i)\frac{N_i^{n-1}(u)}{u_{i+n} - u_i} + (u_{i+n+1} - u)\frac{N_{i+1}^{n-1}(u)}{u_{i+n+1} - u_{i+1}}$$

where:

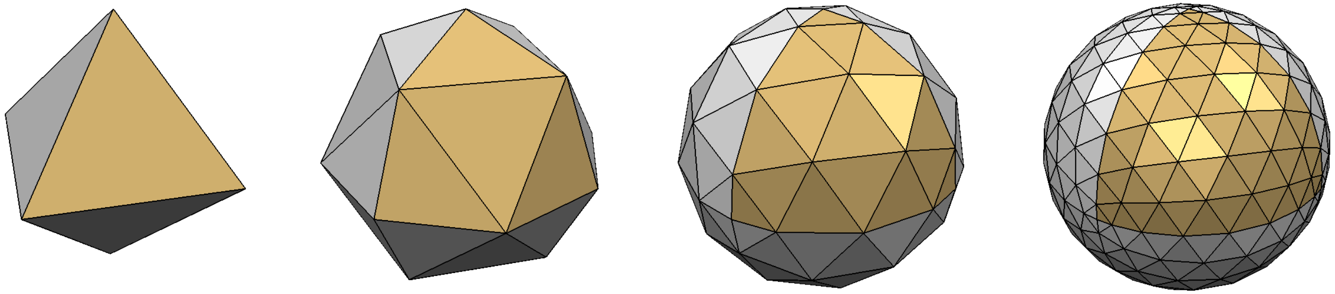
$$N_i^0(u) = \begin{cases} 1, & u \in [u_i, u_{i+1}] \\ 0, & else \end{cases}$$

Curves

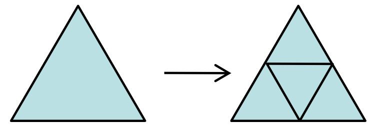
- Bezier and B-Spline curves usually don’t interpolate exactly the control points
- Different from natural cubic splines
- Both Bezier and B-Spline curves are widely used

Subdivision Surfaces

- Generalization of spline curves / surfaces
  - Arbitrary control meshes
  - Successive refinement (subdivision)
  - Converges to smooth limit surface
  - Connection between splines and meshes

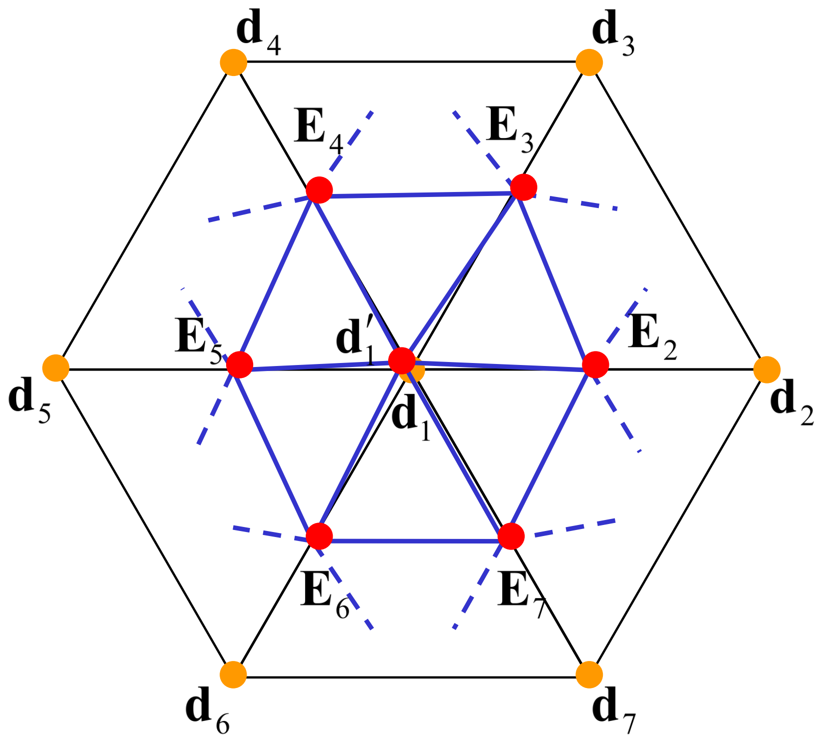


Loop Subdivision



- Generalization of *box splines*
- Primal, approximating subdivision scheme
- Applied to *triangle* meshes
- Generates *G^2 continuous* limit surfaces:
  – C^1 for the set of finite extraordinary points
    - Vertices with valence $\neq 6$
  – C^2 continuous everywhere else

Loop Subdivision

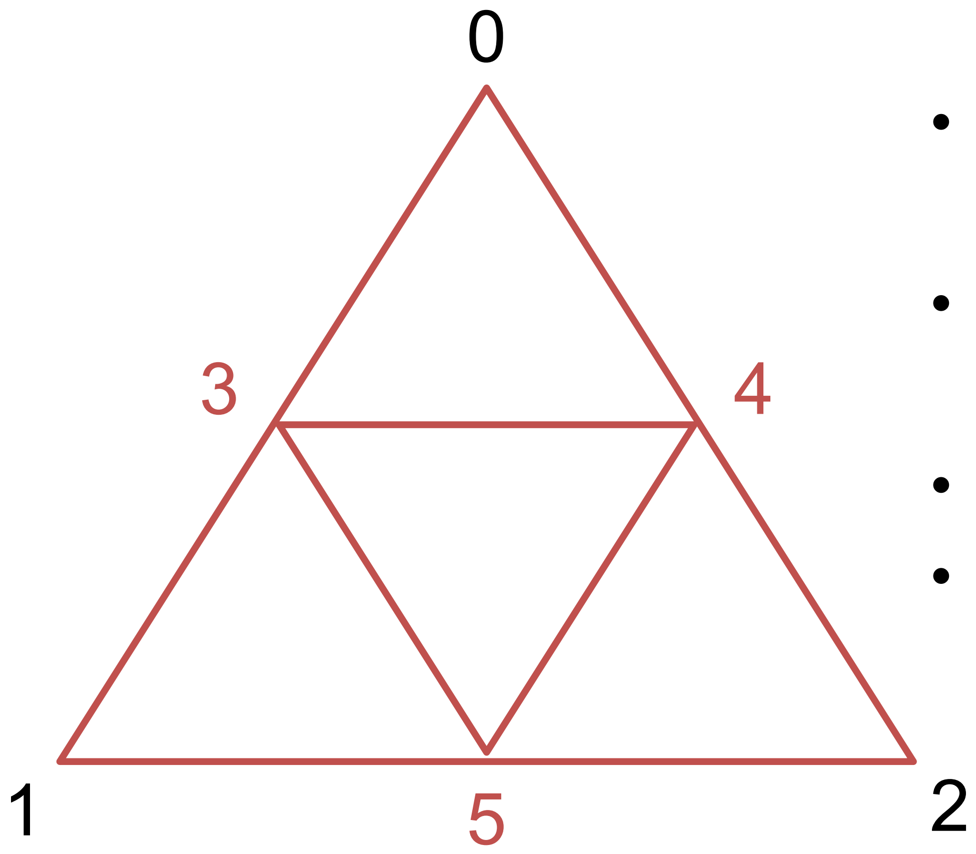


$$E_i = \frac{3}{8}(d_1 + d_i) + \frac{1}{8}(d_{i-1} + d_{i+1})$$

$$\mathbf{d}_1' = \alpha_n \mathbf{d}_1 + \frac{(1-\alpha_n)}{n} \sum_{j=2}^{n+1} \mathbf{d}_j$$

$$\alpha_n = \frac{3}{8} + \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2$$

Loop Subdivision



- Mesh represented with vertex positions (implicit indices) and faces
- Every face in the old mesh become 4 new faces in the subdivided mesh
- Avoid duplicates
- Adjacency / connectivity information is needed

Reference

- libigl develop by Interactive Geometry Lab lead by Prof. Dr. Olga Sorkine-Hornung
- OpenMesh library uses a halfedge data structure
- OpenSubdiv by Pixar implements high performance subdivision surface evaluation on massively parallel CPU and GPU architectures