Light and Colors

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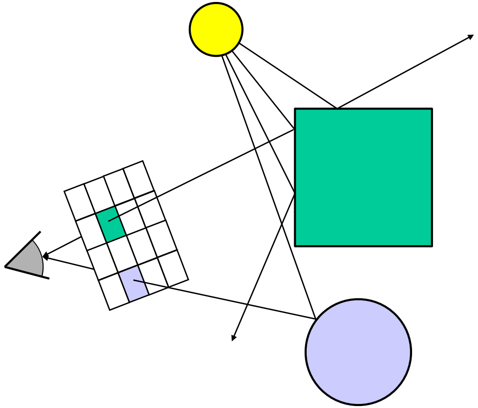
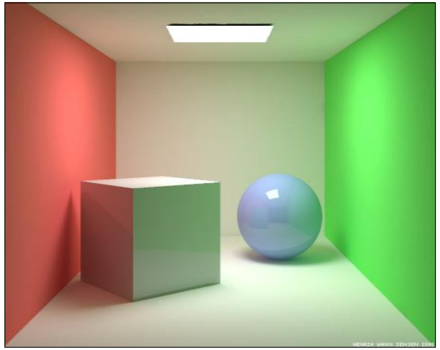
cgl

Light in Computer Graphics

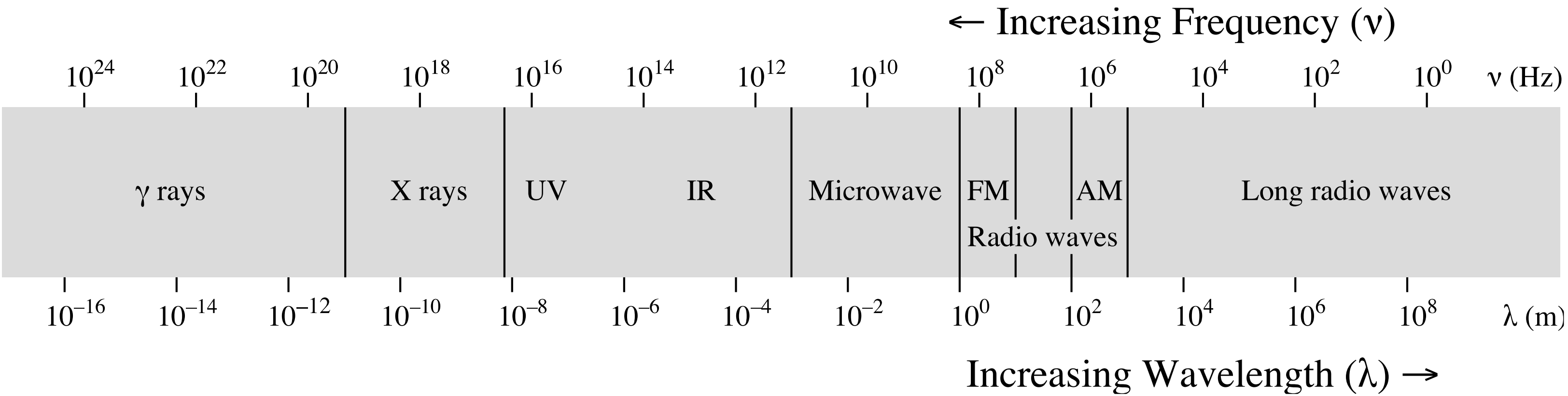




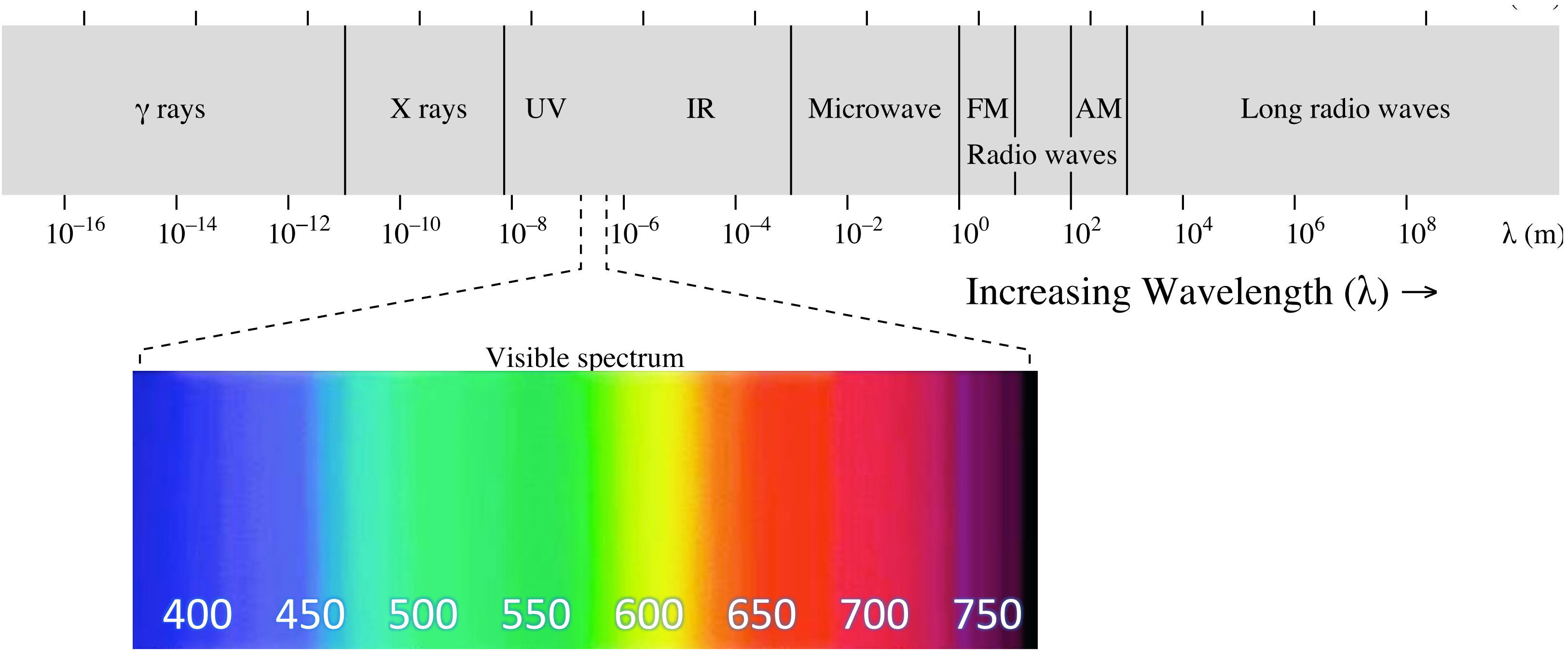
- Computer graphics “=” generating images
- Image = array of pixels
- Each pixel represents one light ray (or more)

What is light?

- A form of electromagnetic (EM) radiation
  - x-rays, microwaves, radio waves, …
  - Amplitude determines intensity
- We perceive a limited section of the spectrum as “visible light”

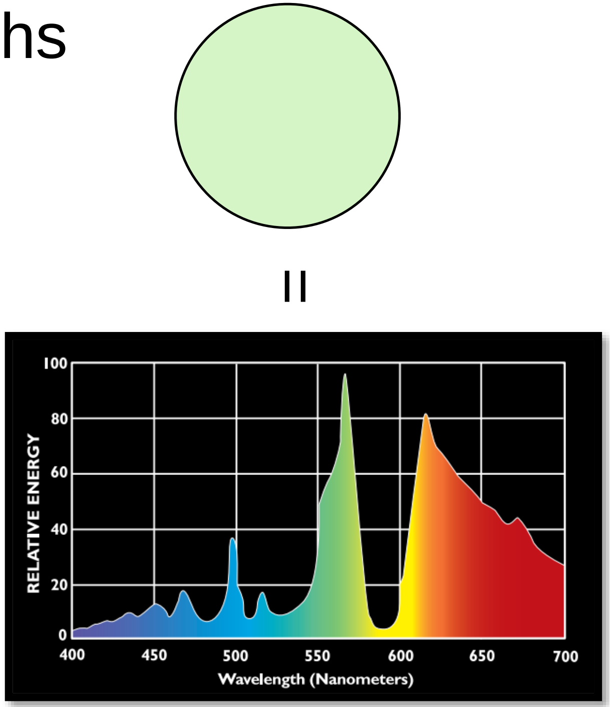


What is color?



Spectral Distribution of Illumination

- Light can be a mixture of many wavelengths
- Spectral power distribution (SPD)
  - P(λ) = intensity at wavelength λ
  - intensity as a function of wavelength
- We perceive these distributions as colors



Measuring Light

- Each ray carries a spectrum P(λ)
- P(λ) contains more information than humans *can* and *need to* process
- Humans "project" this spectrum onto a lower-dimensional subspace

Review of 3D Vector Spaces

- $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ orthonormal basis vectors

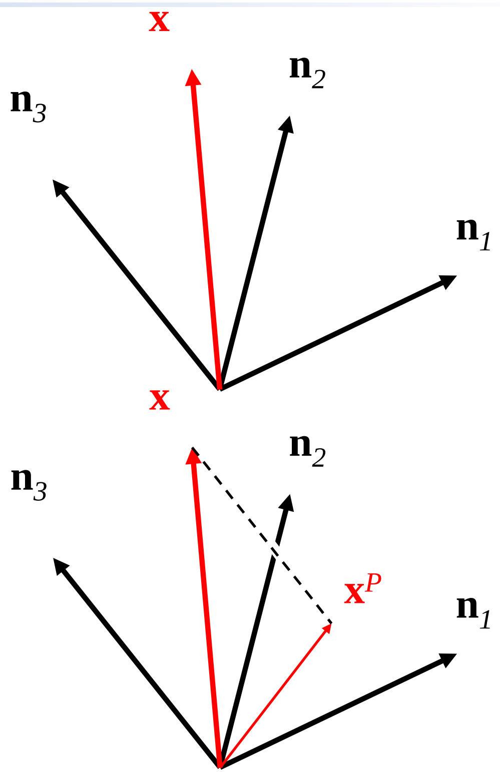
$$\mathbf{x} = x_1\mathbf{n}_1 + x_2\mathbf{n}_2 + x_3\mathbf{n}_3$$

- Coordinates are inner products

$$\mathbf{x} = (\mathbf{x} \cdot \mathbf{n}_1)\mathbf{n}_1 + (\mathbf{x} \cdot \mathbf{n}_2)\mathbf{n}_2 + (\mathbf{x} \cdot \mathbf{n}_3)\mathbf{n}_3$$

- Projection onto 2D subspace

$$\mathbf{x}^P = (\mathbf{x} \cdot \mathbf{n}_1)\mathbf{n}_1 + (\mathbf{x} \cdot \mathbf{n}_2)\mathbf{n}_2$$



Infinite Dimensional Space

- Infinite dimensional vector is a function

$$\mathbf{x}^{\mathrm{3D}} = (x_1, x_2, x_3) \qquad \longrightarrow \qquad \mathbf{x}^{\mathrm{inf}} = x(\lambda)$$

- Infinite number of basis functions needed
- Projection onto 3D subspace with $n_1(\lambda)$, $n_2(\lambda)$, $n_3(\lambda)$ orthonormal basis functions

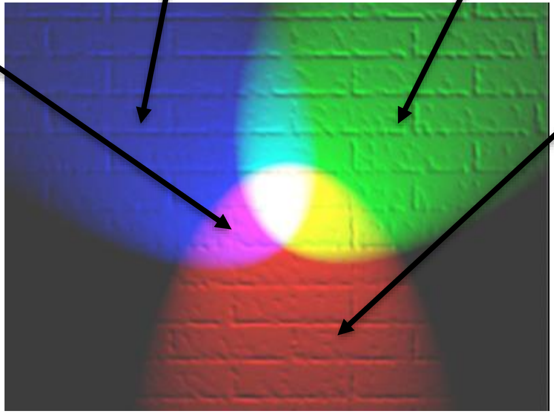
$$\mathbf{x}^{\mathrm{P}}(\lambda) = x_1 n_1(\lambda) + x_2 n_2(\lambda) + x_3 n_3(\lambda)$$

- Coordinates are continuous inner products: $x_i = \int x(\lambda) n_i(\lambda) d\lambda$

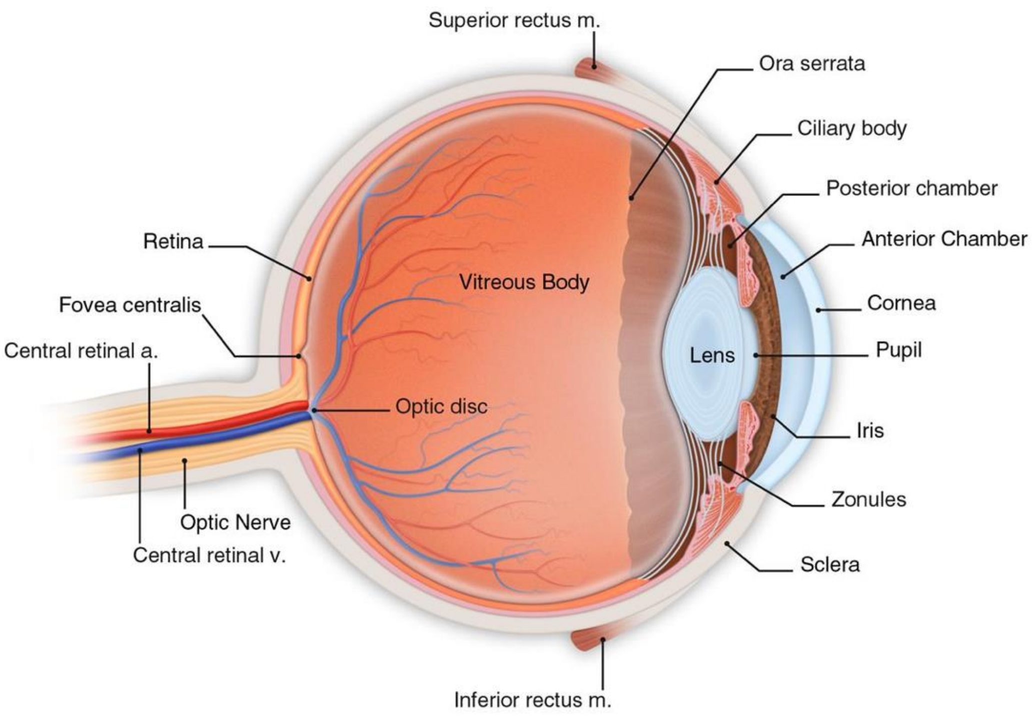
What is color?

- Perception of light of certain wavelengths
- Can combine primary colors

$$\mathbf{x}^{\mathrm{P}}(\lambda) = x_1 n_1(\lambda) + x_2 n_2(\lambda) + x_3 n_3(\lambda)$$

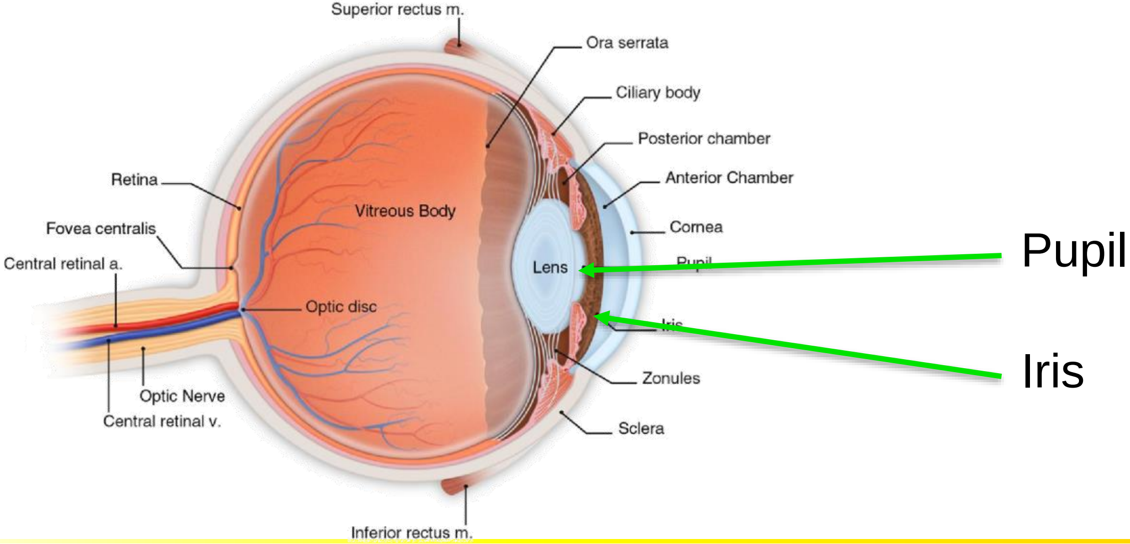


Anatomy of the Eye



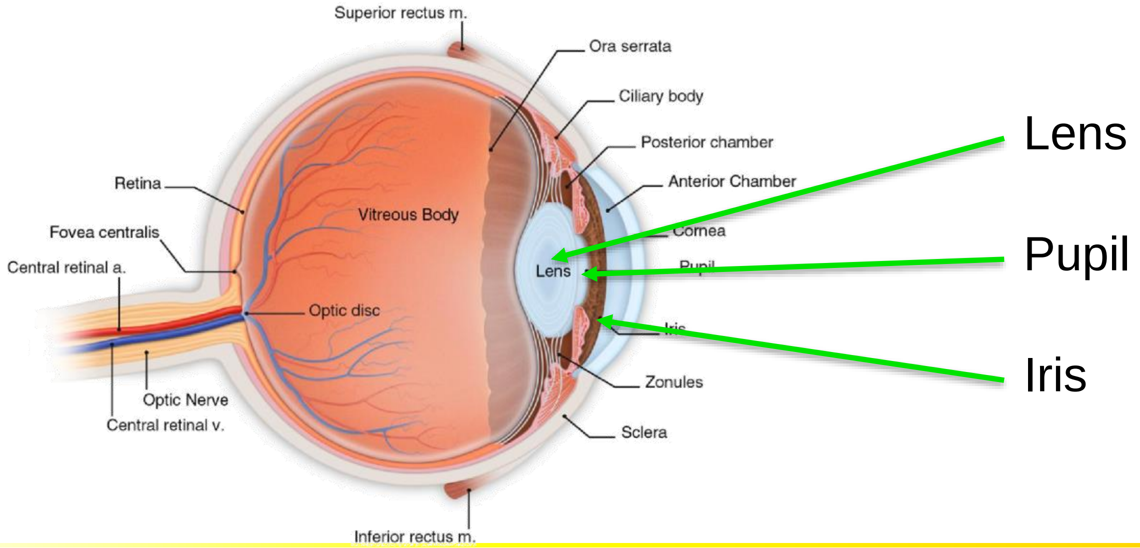
Anatomy of the Eye

- **Iris** lets light into eye
  - contracts and dilates in response to brightness
  - the hole in the iris is the **pupil**



Anatomy of the Eye

- **Lens** focuses light on retina
  - dynamically reshaped by surrounding muscles to control focus



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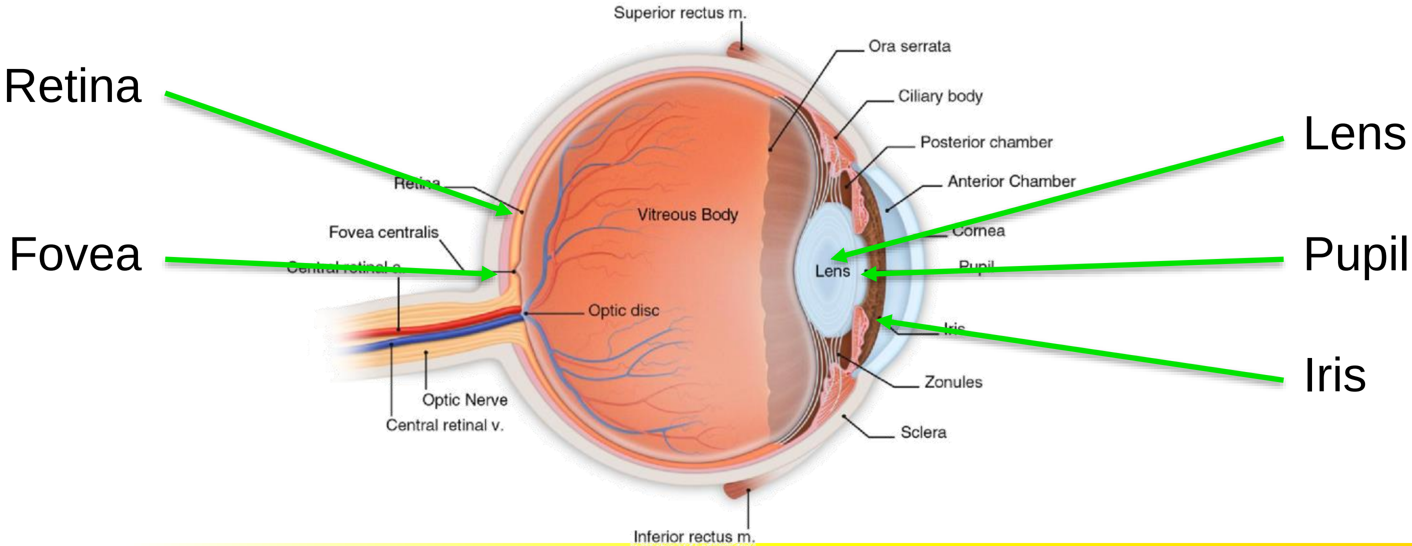
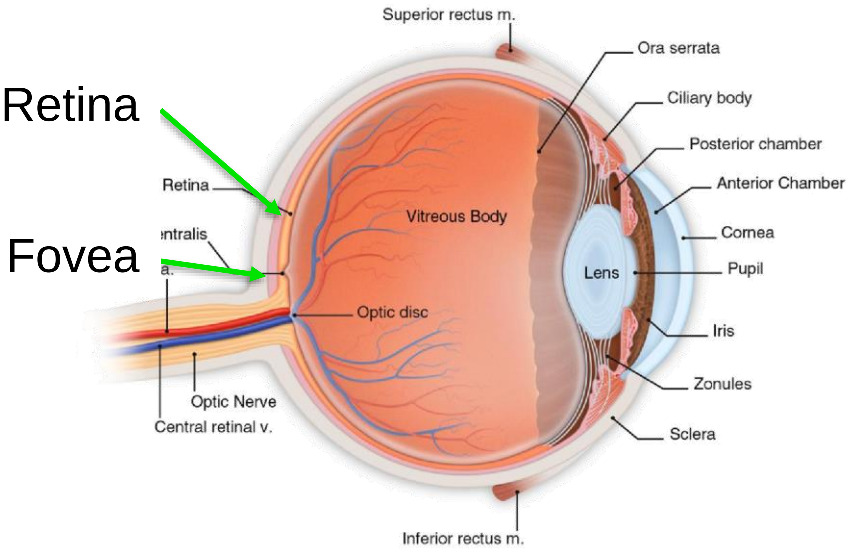
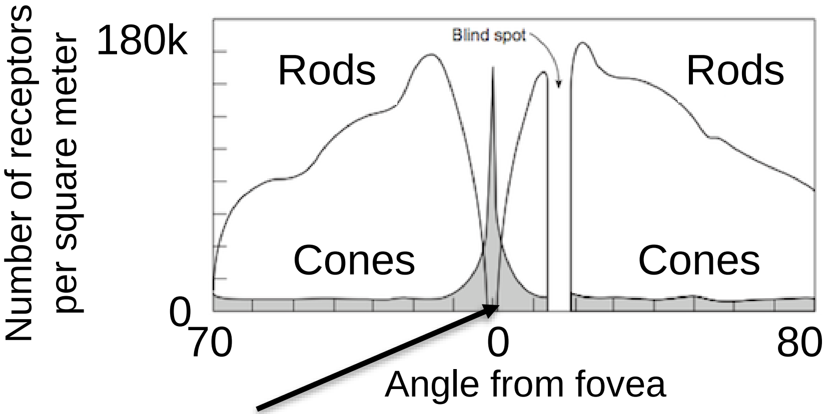
Anatomy of the Eye

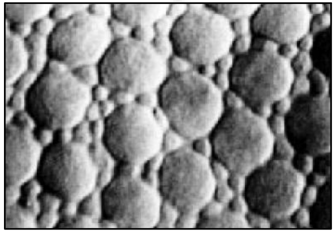
- Cells in **retina** react to light
  - sends signals via optic nerve to brain
  - **fovea** is the region of highest acuity

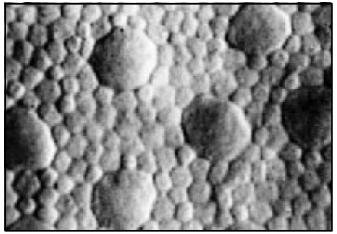


Retinal Composition : Two Kinds of Cells



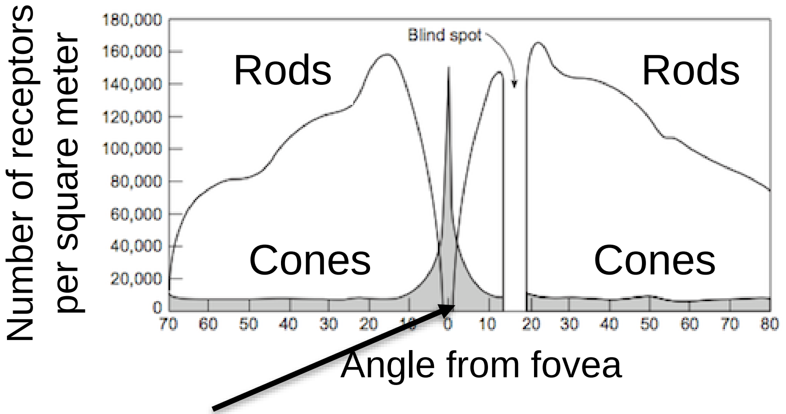


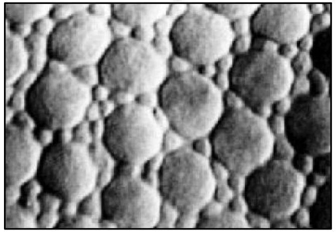

near fovea


away from fovea

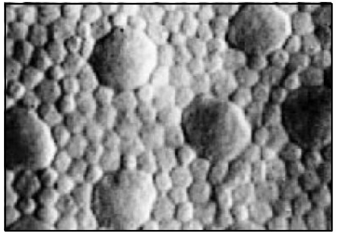
Retinal Composition : Two Kinds of Cells

- **Cones** are concentrated in fovea
  - high acuity, require more light
  - respond to color
- **Rods** concentrated outside fovea
  - lower acuity, require less light
  - respond to intensity only





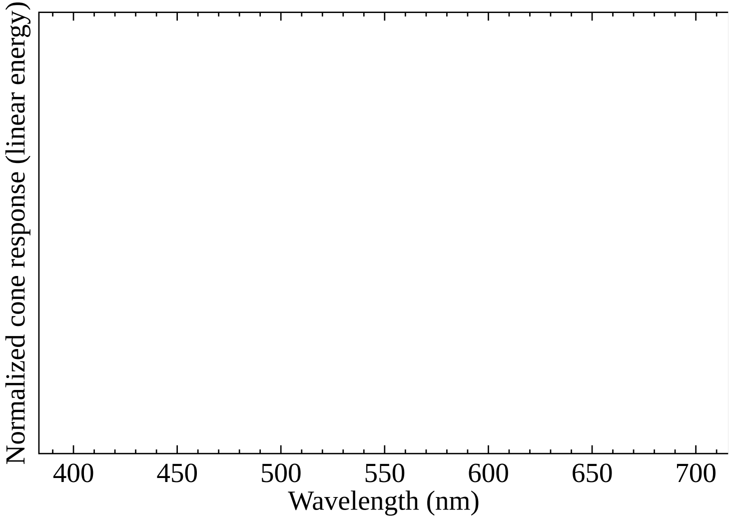
near fovea



away from fovea

The Response of Cones to Color

- Three kinds of cones: S, L, and M
  - S: short-wavelengths (“blue”)
  - M: medium-wavelengths (“green”)
  - L: long-wavelengths (“red”)



- Eye projects P(λ) into 3D subspace using these three basis functions/vectors

Color Perception

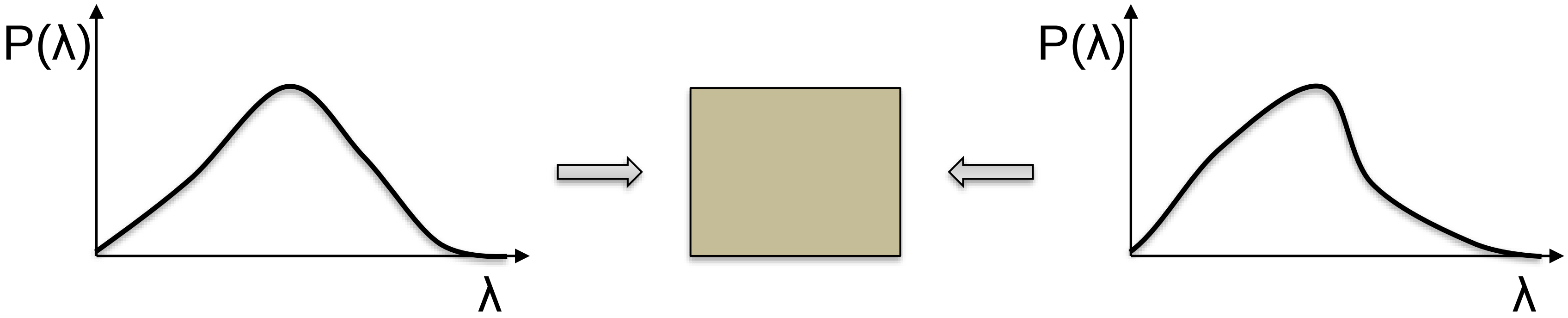
- Humans project P(λ) into a 3D subspace
- Most mammals have 2 types of cones (2D subspace)
- Whales, dolphins, among other sea animals, have a single type of cone

Color Perception

- Many birds have UV receptors, some can see magnetic fields
- Some animals have more than 3 cones:
  - Mantis Shrimp use an 8D subspace!

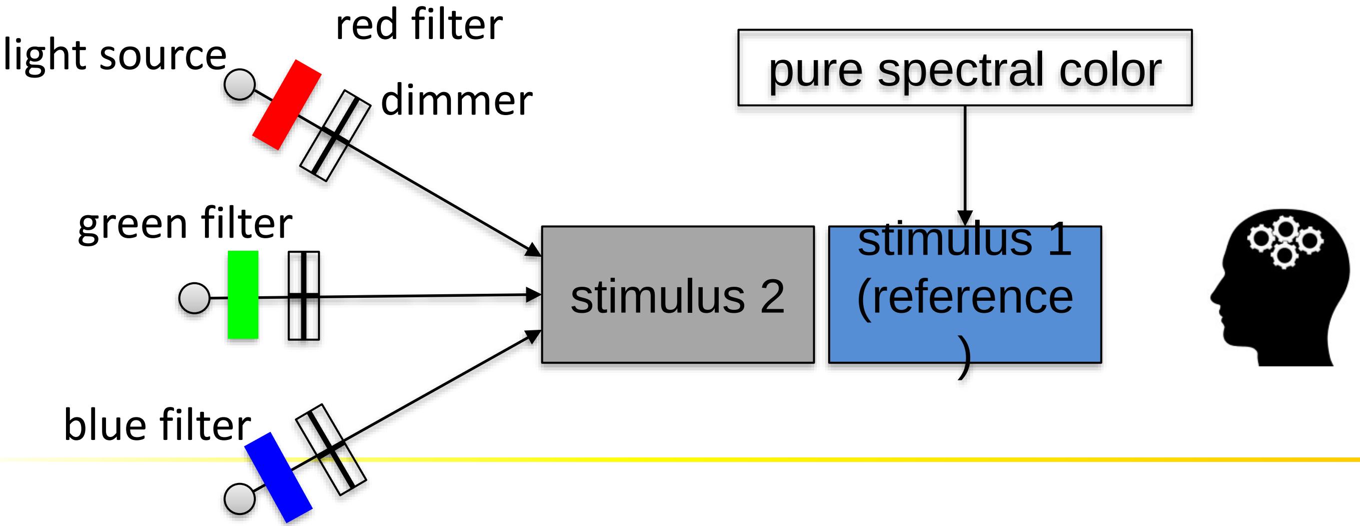
Metamers

- We project infinite dimensional space onto 3D
- Some information must be lost!
- Two completely different SPDs might look the same to us



The CIE Primary System (1931)

- **C**ommission **I**nternationale de l’**E**clairage
- Setup for measuring human color sensitivity
  - Three light sources at: 435.8, 546.1, and 700.0 nm



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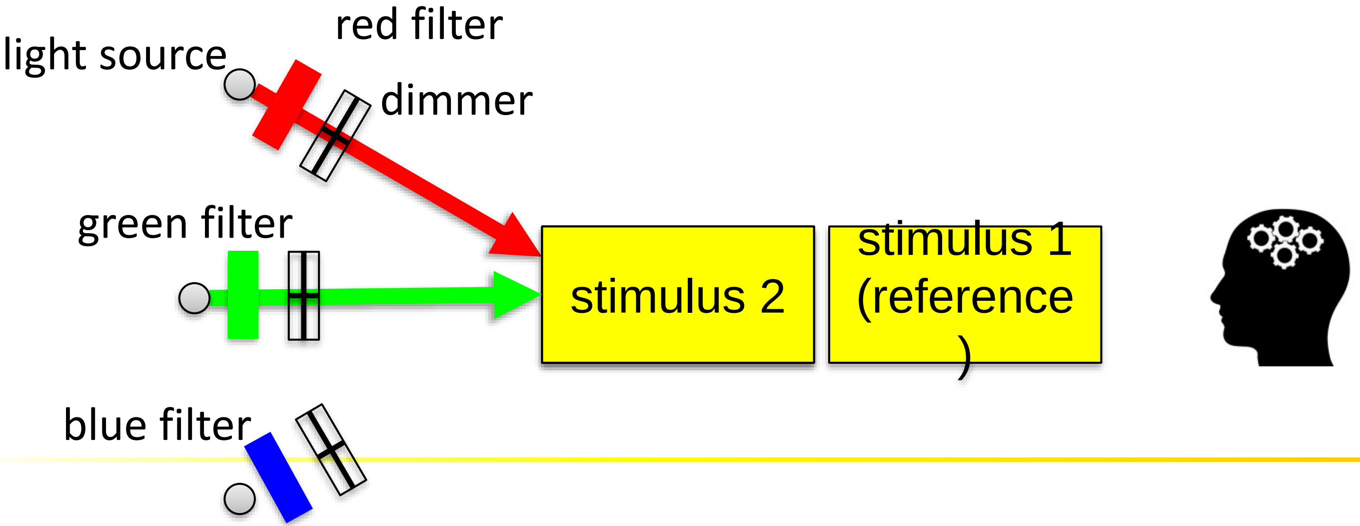
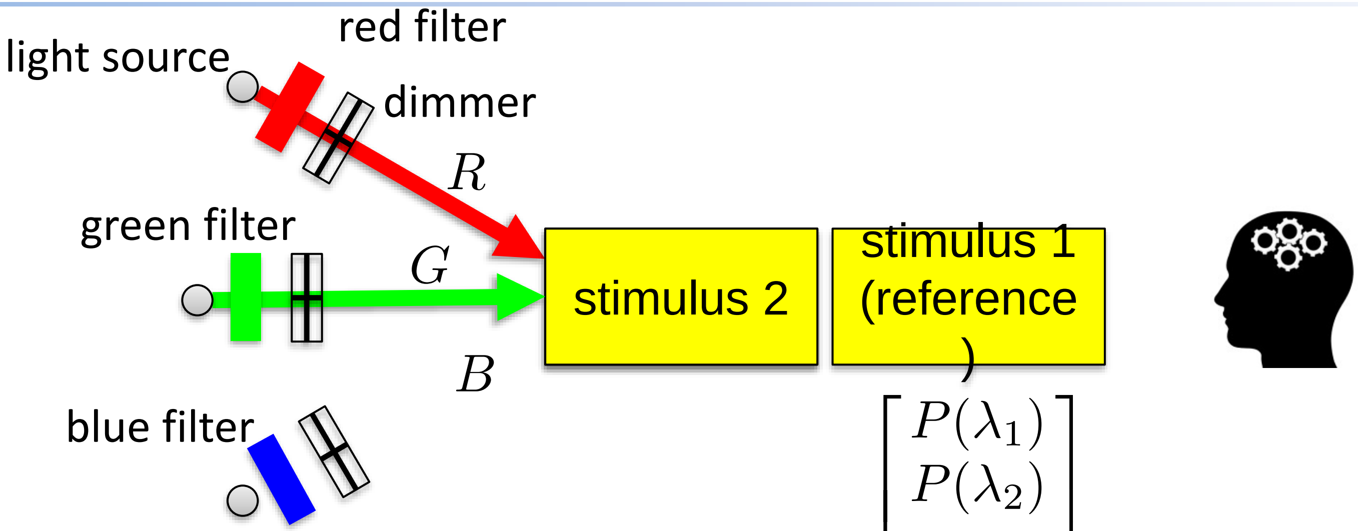


The CIE Primary System (1931)



$$\begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} P(\lambda_1) \\ P(\lambda_2) \\ \vdots \\ P(\lambda_N) \end{bmatrix}$$

Color Matching as Matrix Multiplication

Intensities for the three primary lights — Color matching functions — SPD of test light

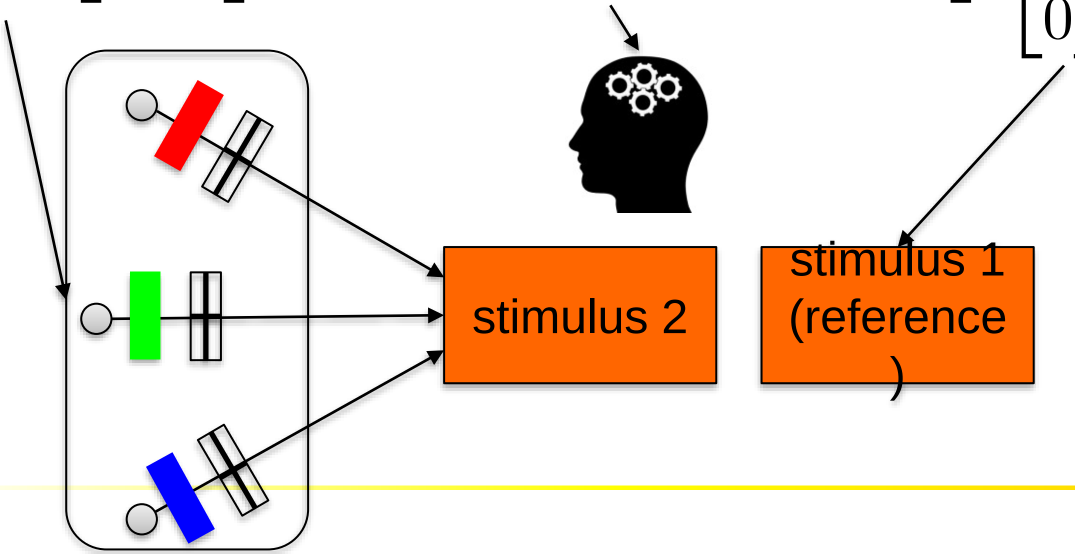
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} \bar{r}(\lambda_1) & \bar{r}(\lambda_2) & \dots & \bar{r}(\lambda_N) \\ \bar{g}(\lambda_1) & \bar{g}(\lambda_2) & \dots & \bar{g}(\lambda_N) \\ \bar{b}(\lambda_1) & \bar{b}(\lambda_2) & \dots & \bar{b}(\lambda_N) \end{bmatrix} \begin{bmatrix} P(\lambda_1) \\ P(\lambda_2) \\ \vdots \\ P(\lambda_N) \end{bmatrix}$$

$$\begin{bmatrix} \bar{r}(\lambda_1) \\ \bar{g}(\lambda_1) \\ \bar{b}(\lambda_1) \end{bmatrix} = \begin{bmatrix} \bar{r}(\lambda_1) & \bar{r}(\lambda_2) & \dots & \bar{r}(\lambda_N) \\ \bar{g}(\lambda_1) & \bar{g}(\lambda_2) & \dots & \bar{g}(\lambda_N) \\ \bar{b}(\lambda_1) & \bar{b}(\lambda_2) & \dots & \bar{b}(\lambda_N) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

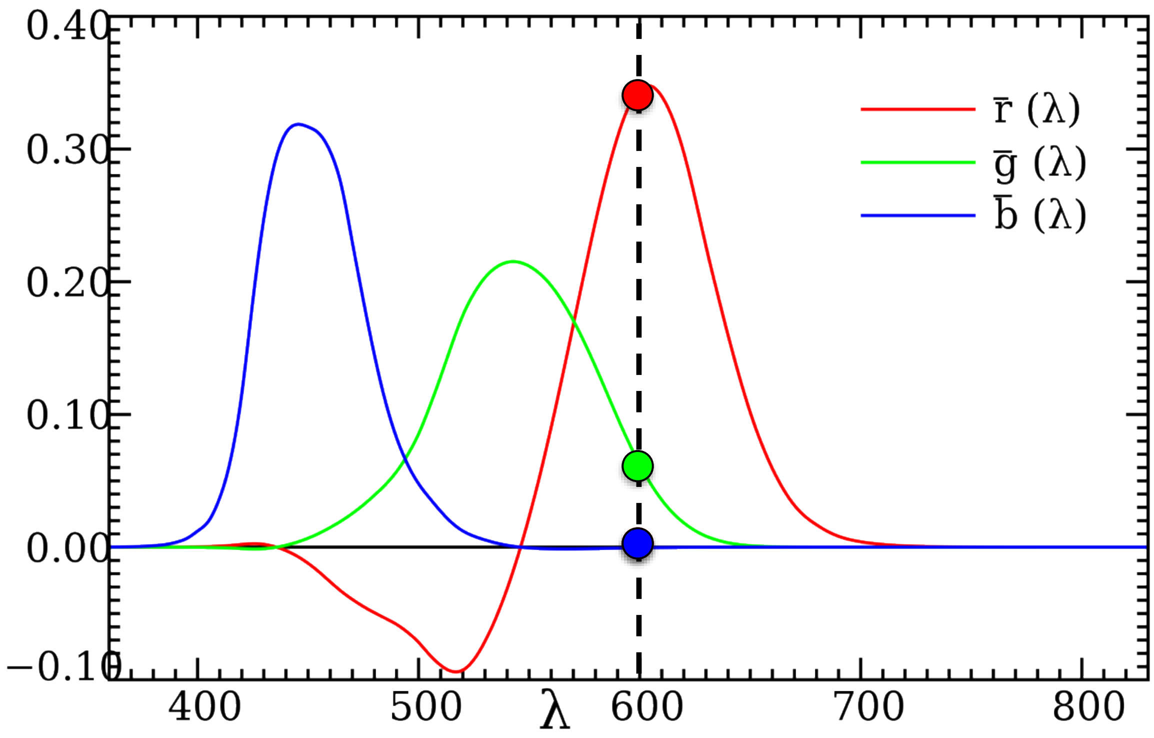
Monochromatic test light

Color Matching as Matrix Multiplication

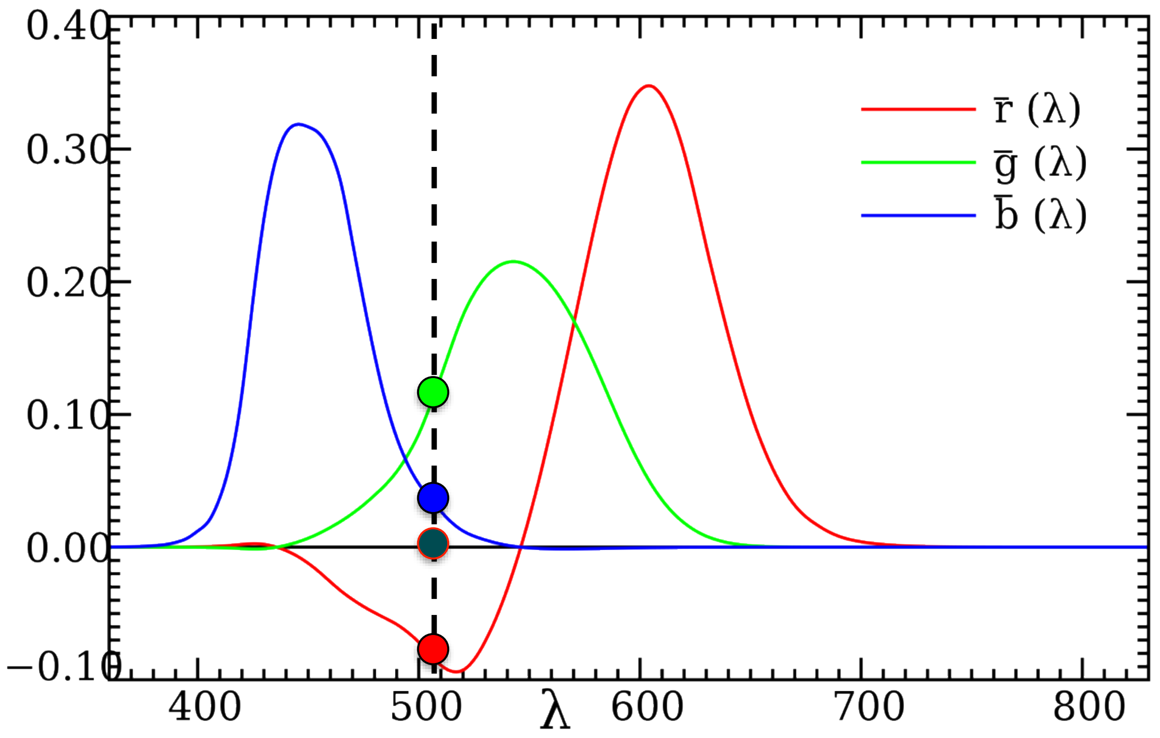
$$\begin{bmatrix} \bar{r}(\lambda_1) \\ \bar{g}(\lambda_1) \\ \bar{b}(\lambda_1) \end{bmatrix} = \begin{bmatrix} \bar{r}(\lambda_1) & \bar{r}(\lambda_2) & \dots & \bar{r}(\lambda_N) \\ \bar{g}(\lambda_1) & \bar{g}(\lambda_2) & \dots & \bar{g}(\lambda_N) \\ \bar{b}(\lambda_1) & \bar{b}(\lambda_2) & \dots & \bar{b}(\lambda_N) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$



CIE 1931 RGB Color Matching Functions

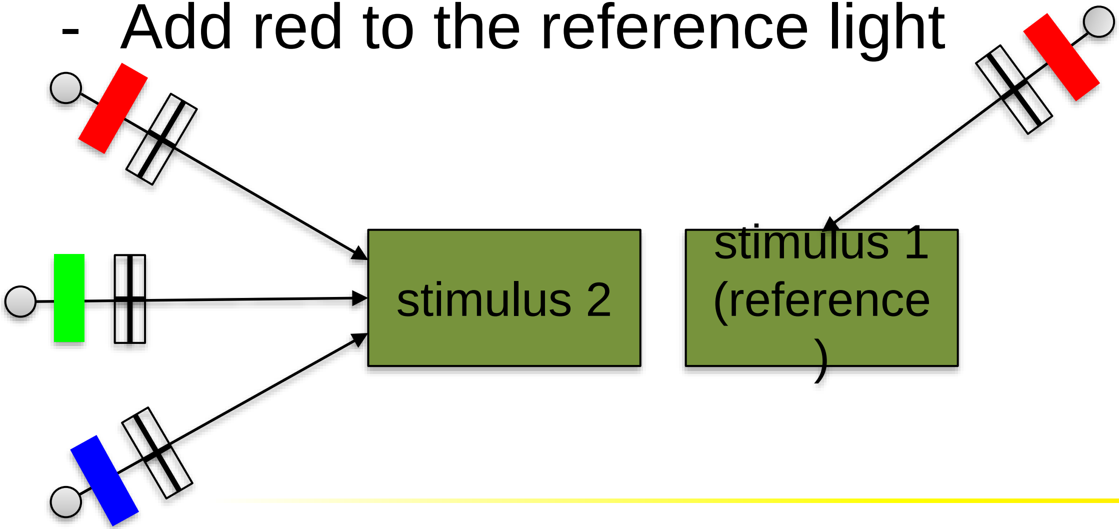


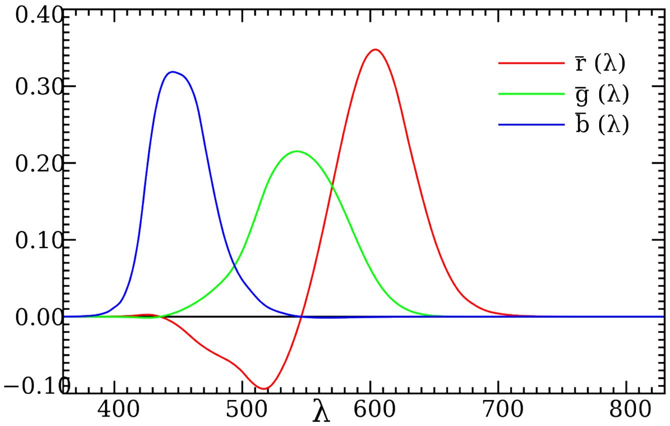
CIE 1931 RGB Color Matching Functions



Negative Matching Values?

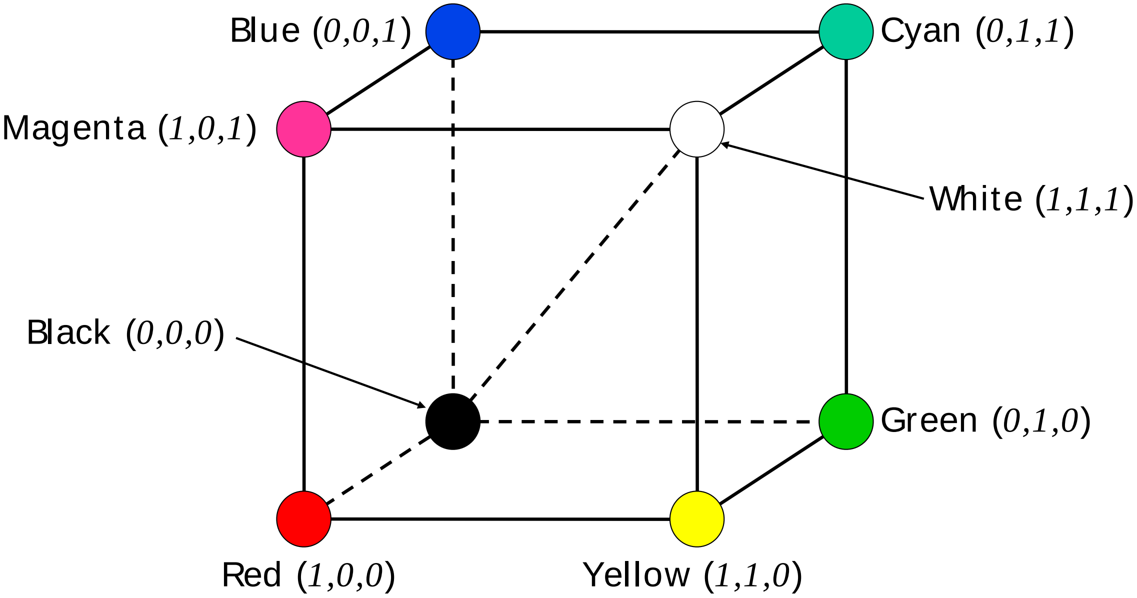
- What do these negative values mean?
  - Some colors **cannot** be written as a combination of red, green and blue!
  - Add red to the reference light





RGB Color Space

- Unit cube with R,G,B basis vectors



Other Color Spaces

- Our choice of RGB color space is fairly arbitrary, based on our perceptual system

- We could in principle select any 3 primaries
  - different basis vectors, linear transformation
  - new basis spans same 3D subspace

- We can also construct other 3D color spaces

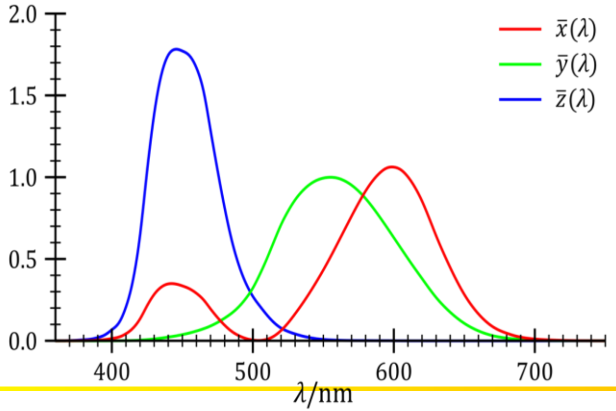
CIE XYZ Color Space

- Infinitely many ways to obtain non-negative matching functions!
- Lets call ours XYZ
- Represents all perceptible colors
  - Vector (X, Y, Z) quantifies any spectral color stimulus P(λ) we perceive
  - Compute by inner products of P(λ) with matching functions

$$X = \int_0^\infty P(\lambda)\bar{x}(\lambda)d\lambda$$

$$Y = \int_0^\infty P(\lambda)\bar{y}(\lambda)d\lambda$$

$$Z = \int_0^\infty P(\lambda)\bar{z}(\lambda)d\lambda$$



CIE XYZ Color Space

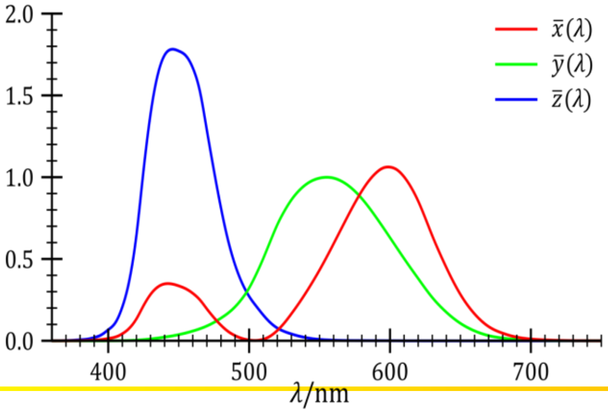
- Linear combinations: XYZ and RGB span the same 3D subspace

$$X = \int_0^\infty P(\lambda)\bar{x}(\lambda)d\lambda$$

$$Y = \int_0^\infty P(\lambda)\bar{y}(\lambda)d\lambda$$

$$Z = \int_0^\infty P(\lambda)\bar{z}(\lambda)d\lambda$$

$$\begin{pmatrix}\bar{x}(\lambda)\\ \bar{y}(\lambda)\\ \bar{z}(\lambda)\end{pmatrix} = \begin{pmatrix} 2.36 & -0.515 & 0.005 \\ -0.89 & 1.426 & 0.014 \\ -0.46 & 0.088 & 1.009 \end{pmatrix} \begin{pmatrix}\bar{r}(\lambda)\\ \bar{g}(\lambda)\\ \bar{b}(\lambda)\end{pmatrix}$$



The CIE xyY Color Space

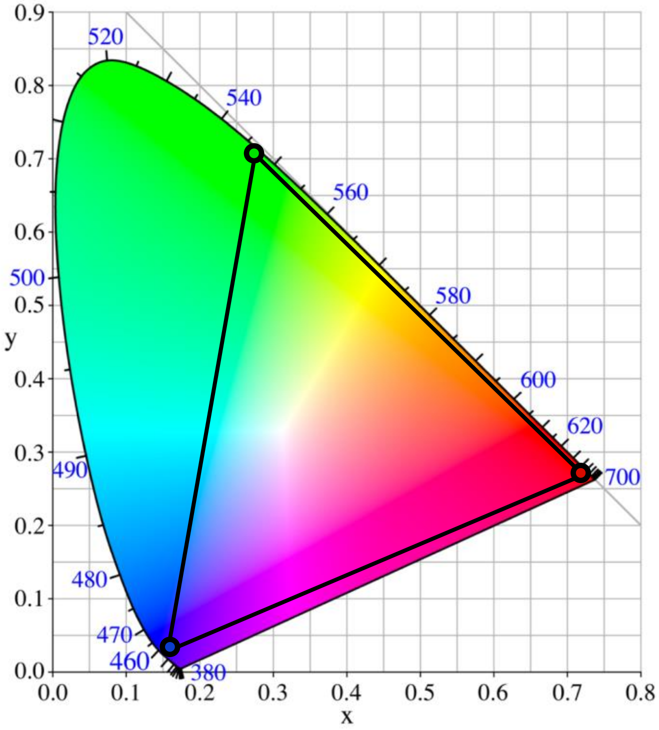
- *Chromaticity* (x,y) can be derived by normalizing the XYZ color components:

$$x = \frac{X}{X+Y+Z} \qquad y = \frac{Y}{X+Y+Z}$$

  - (x,y) characterize *color*
  - Y characterizes *brightness*

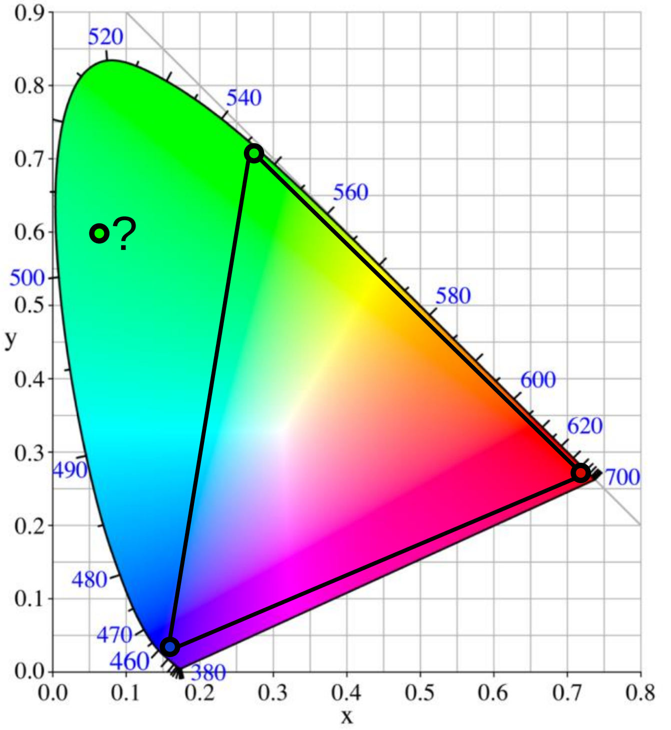
- Plot on xy plane: all colors of a single brightness

CIE Chromaticity Chart



- Primary colors along curved boundary
- Linear combination of two colors : line connecting two points
- Linear combination of 3 colors span a triangle (Color Gamut)

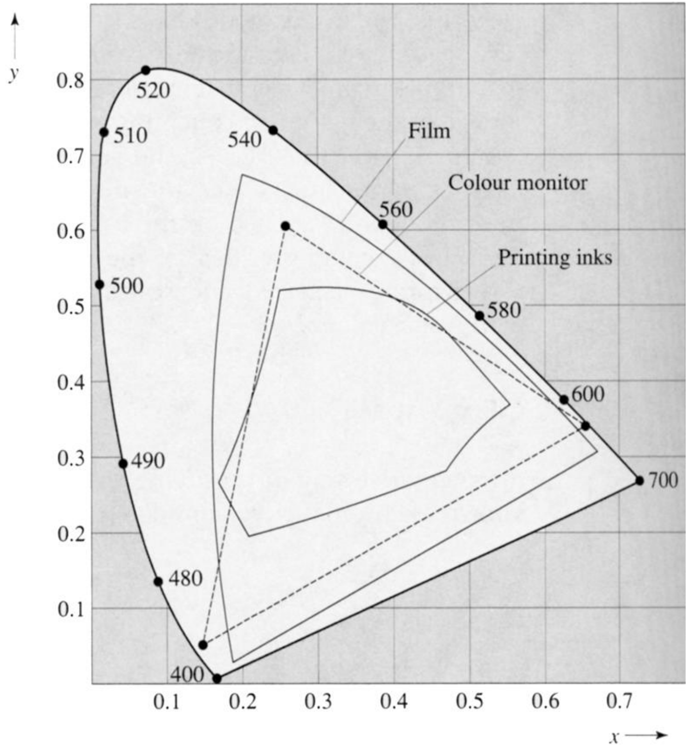
CIE RGB Color Space



- Color primaries at:
- 435.8, 546.1, 700.0 nm

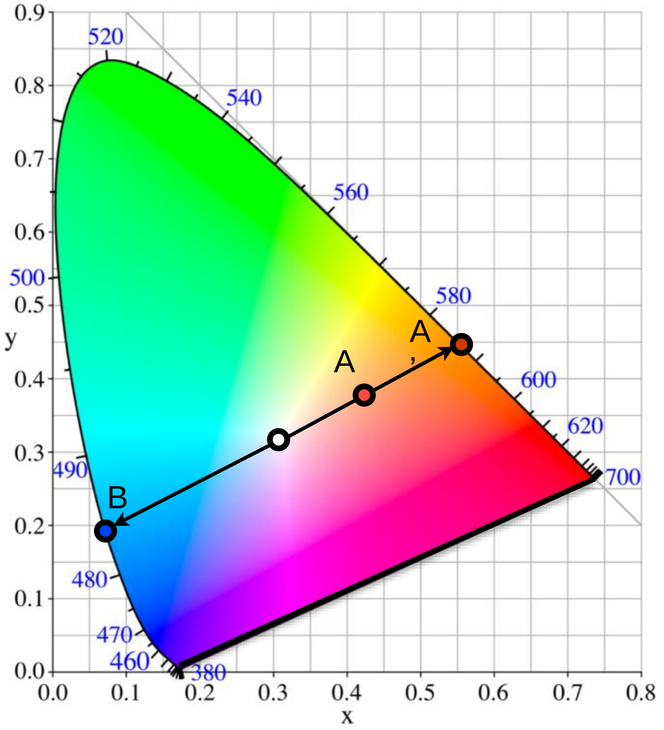
- What about colors outside the gamut?
- How did they appear in the CIE experiment?
- How can we actually plot this diagram on a computer screen?

Color Gamut



y
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
520
510
540
Film
Colour monitor
560
Printing inks
500
580
600
490
700
480
400
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x

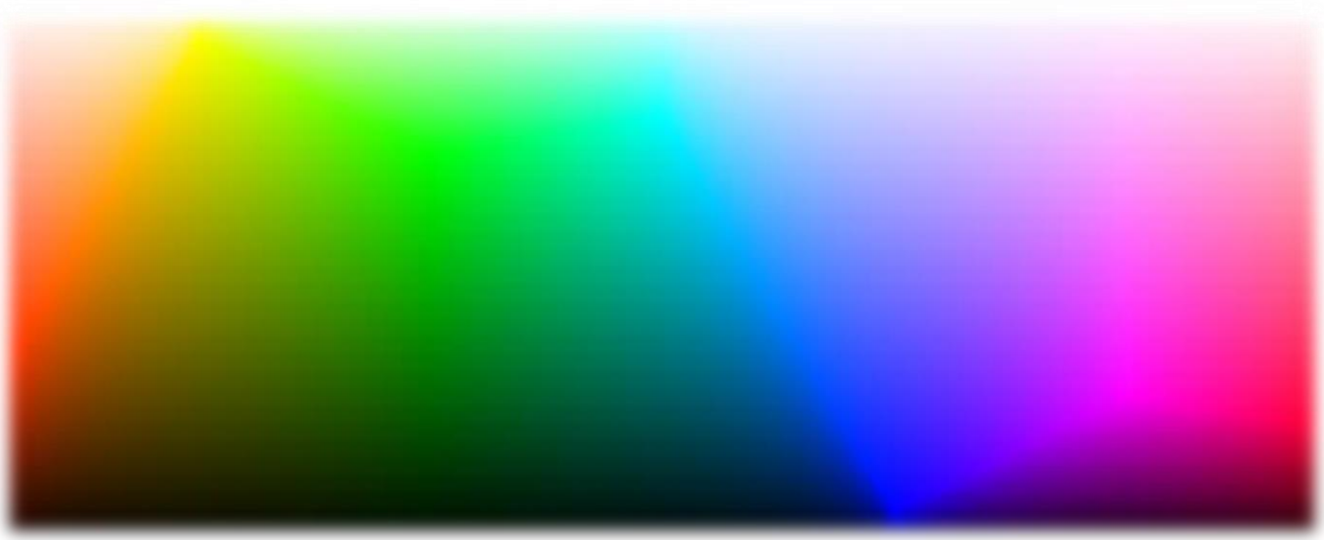
CIE Chromaticity Chart Features



- White Point
- Dominant wavelength
- Inverse color
- Non-spectral purples

Other Color Spaces

- Application specific color spaces for digital representation

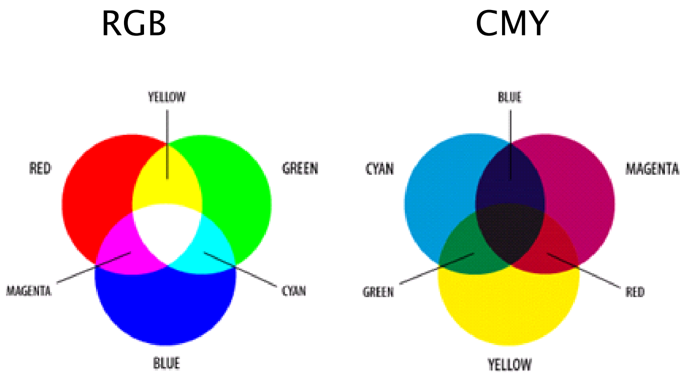


CMY Color Space

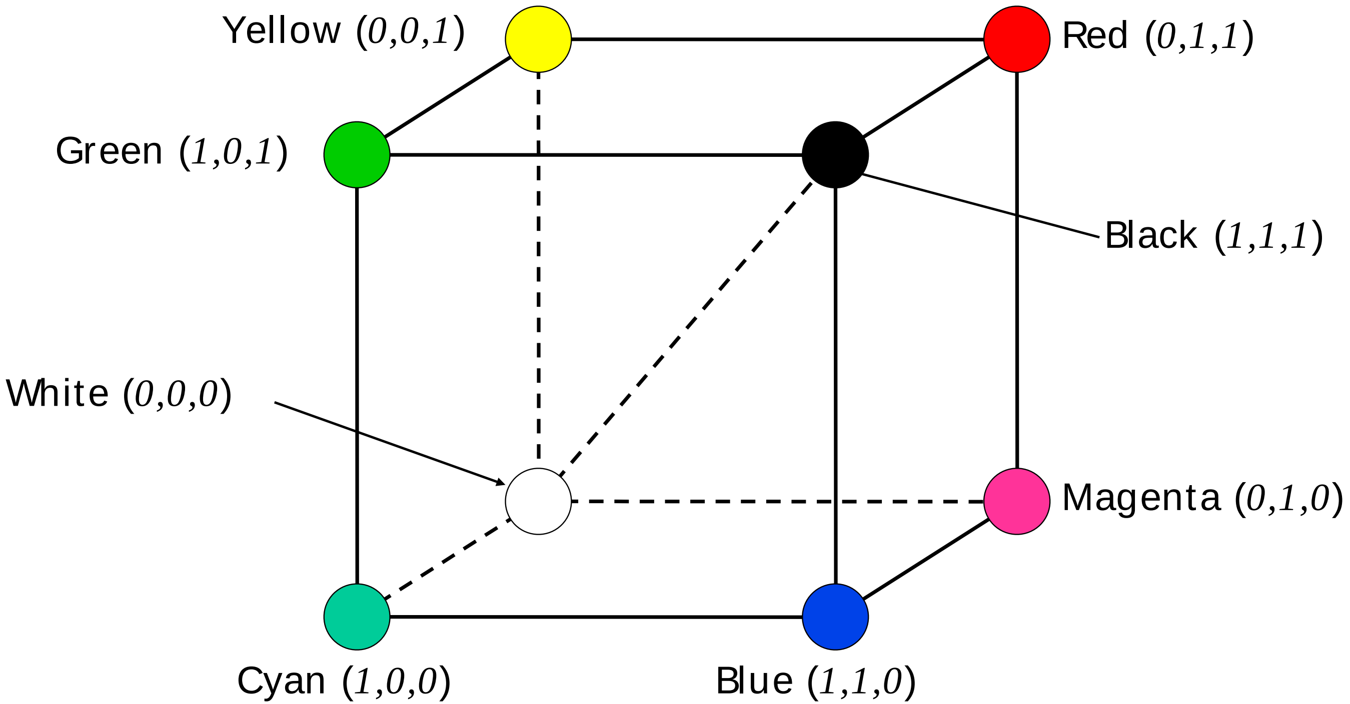
- Used in passive color systems (printers)
- Inverse to RGB
- Transform given by:

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- CMYK: add black as color



CMY Color Space



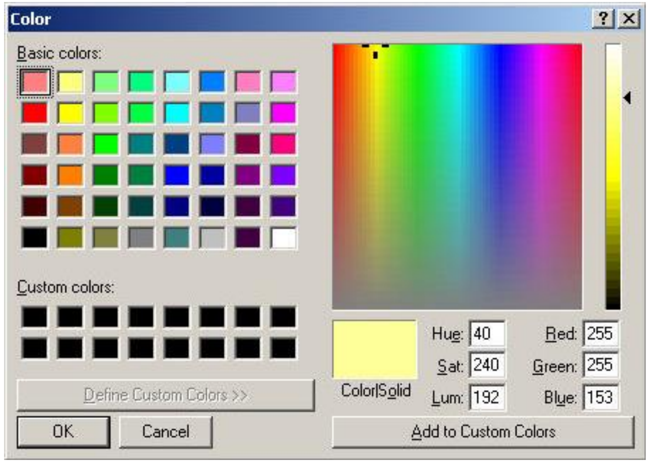
YIQ Color Space

- Luminance Y, In-phase I (orange-blue), Quadrature Q (purple-green) components
- Advantages for natural and skin colors
- NTSC US-color TV standard

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.523 & 0.311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

HSV and HSL/HSB Color Spaces

- User-oriented color spaces
- Intuitive for interactive color picking
- Dimensions no longer primaries:
  - hue: base color
  - saturation: purity of color
  - value/lightness/brightness



- Take RGB, CMY cubes and project to hexagon

HSV and HSL/HSB Color Spaces

- Conversion procedure (RGB→HSV)

```
min = min(R, G, B);
max = max(R, G, B);
V = max;
If (max != 0)
   S = (max - min)/max;
else
   S = 0;
H = Hue (V, S, R, G, B); //procedural
comp.
```

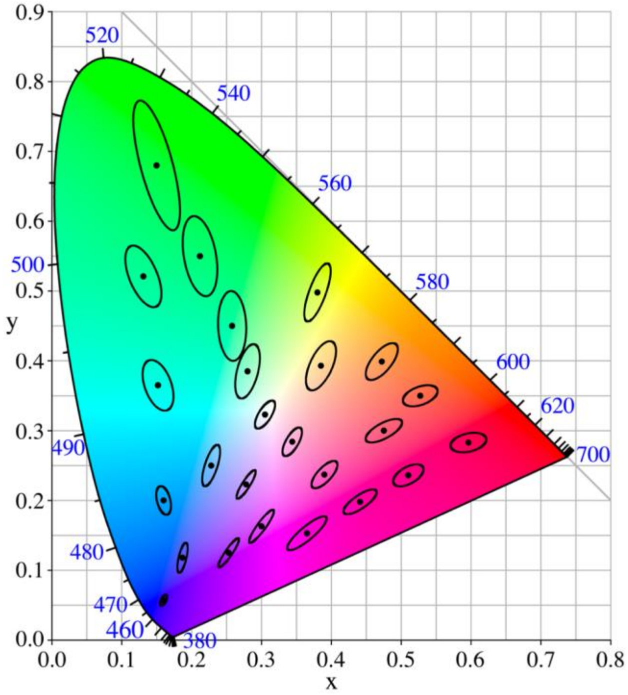
Perceptually-Uniform Color Spaces

- Color spaces so far are perceptually non-uniform:
  - two colors close together in space are not necessarily visually similar
  - two colors far apart are not necessarily very different!

- Measuring “perceptual distance” in color spaces is important

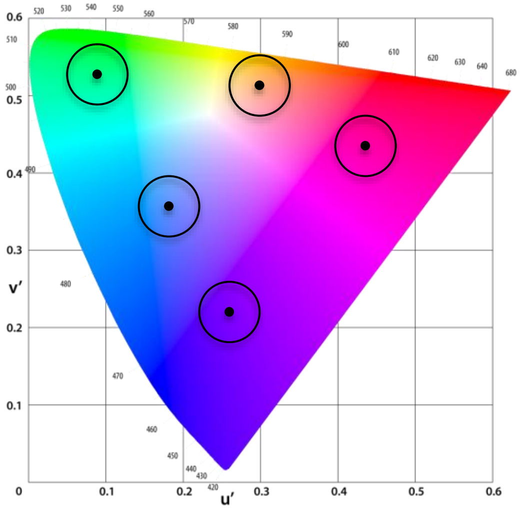
- Experiments by MacAdams

MacAdams Color Ellipses



CIELAB and CIELUV Color Spaces

- MacAdams ellipses become nearly (but not perfectly) circular



OpenGL Color

- 4-vector in vertex- and fragment-shader

```
void main() {
  float r = 1.0;
  float g = 0.7;
  float b = 0.2;
  float a = 1.0;
  gl_FragColor = vec4(r, g, b, a);
}
```

- Normalized to [0,..,1]
- 8 Bits/component -> “true color”

High Dynamic Range (HDR) Imaging

-4 stops

-2 stops

+2 stops

+4 stops

Fused result

Fused result + local tone mapping

Source: wiki commons

END