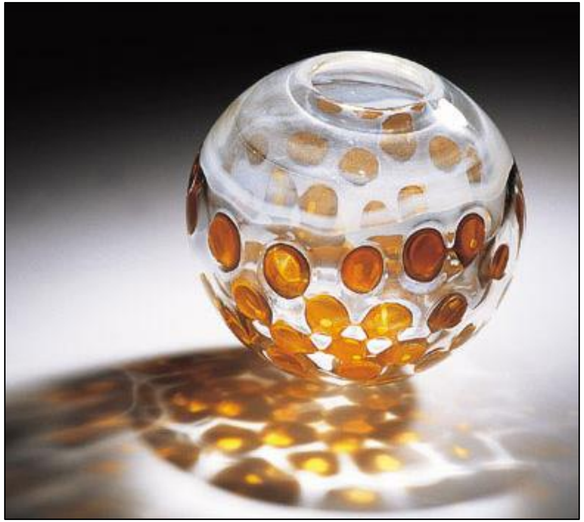
Lighting and Shading I

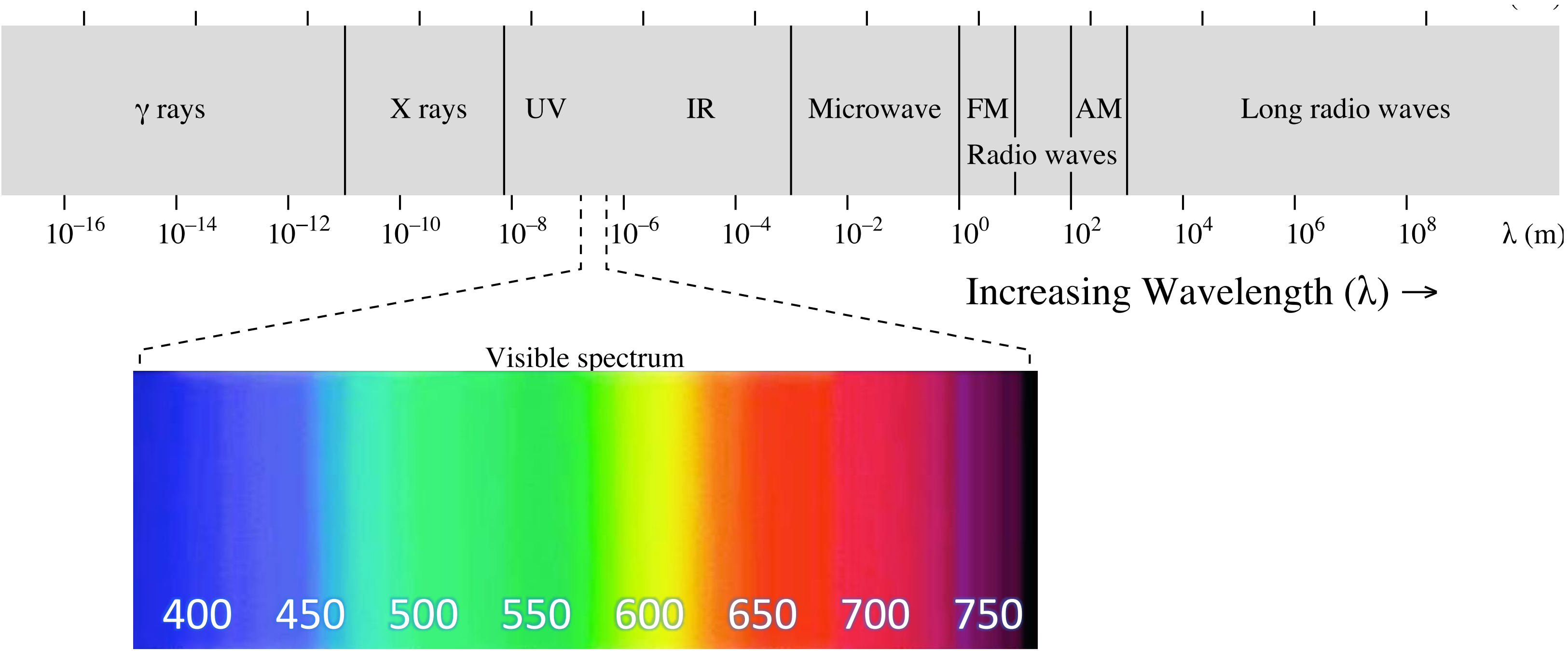
Prof. Dr. Markus Gross



ETH zürich

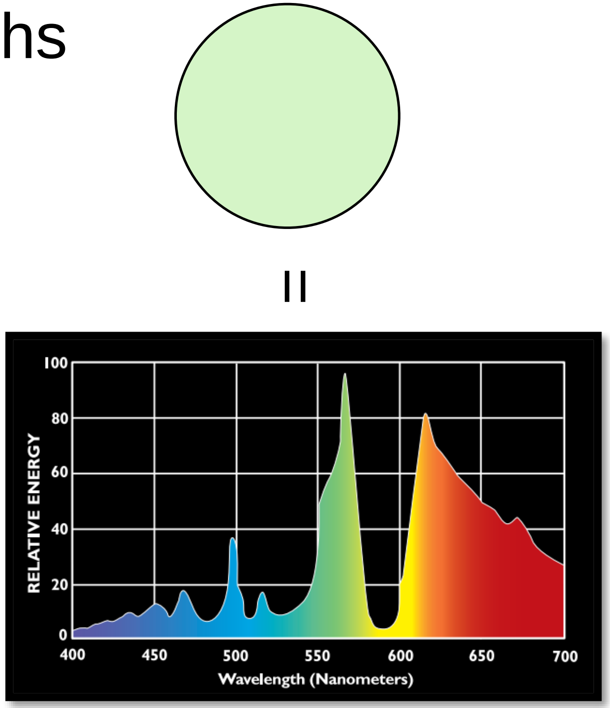
cgl

Before: nature of light and colors



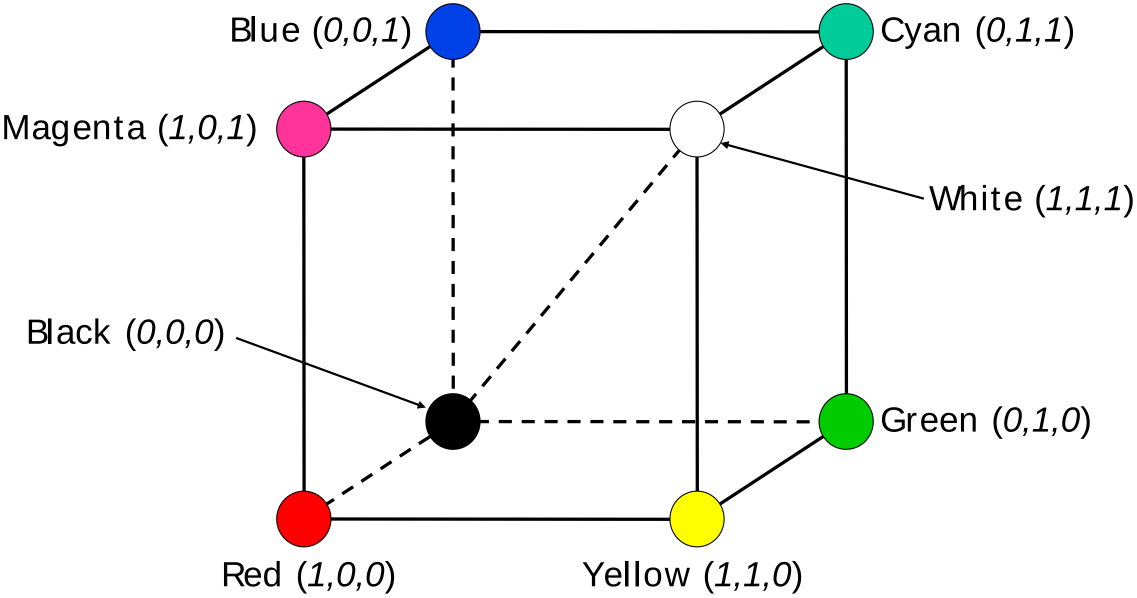
Before: nature of light and colors

- Light can be a mixture of many wavelengths
- Spectral power distribution (SPD)
  - P(λ) = intensity at wavelength λ
  - intensity as a function of wavelength
- We perceive these distributions as colors



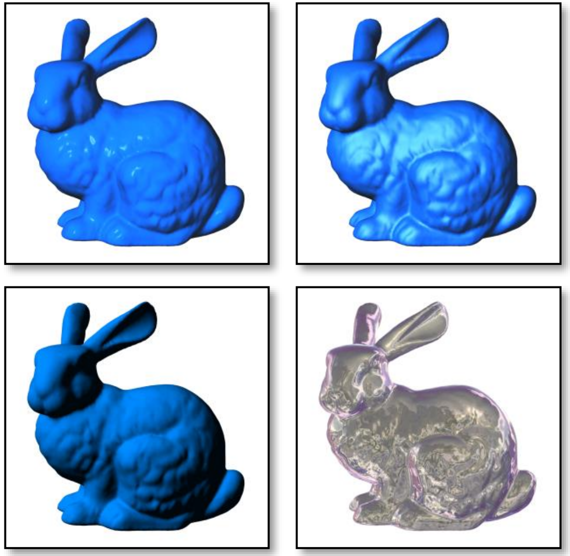
Before: representing colors

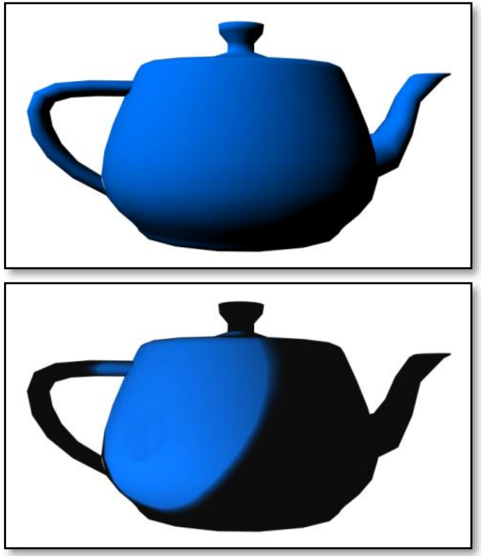
- Unit cube with R,G,B basis vectors



Before: we need material models

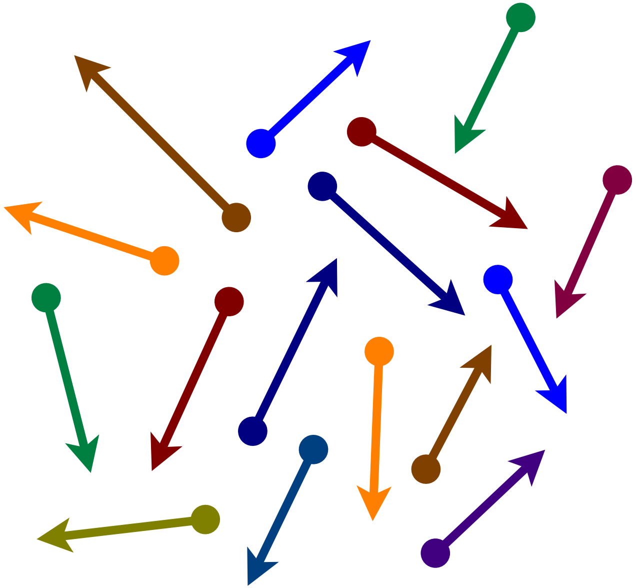
- Interaction of light with geometry





Measuring Light

- How do we measure light



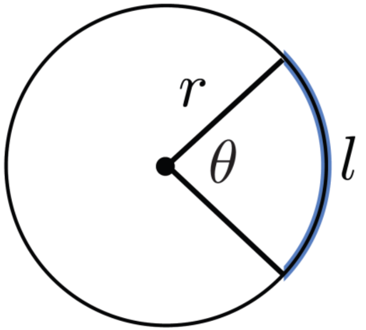
Measuring = Counting photons

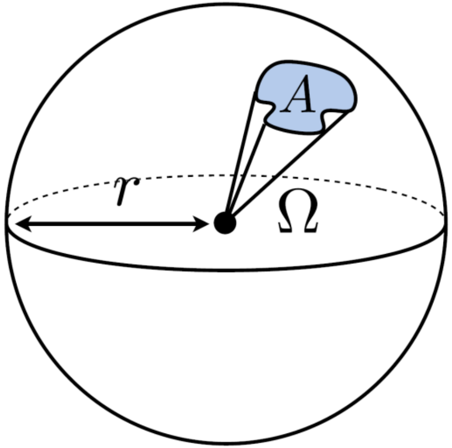
Measuring Light

- Radiometry
  - Studies the measurement of electromagnetic radiation, including visible light

Basic Definitions

- Angle: $\theta = \dfrac{l}{r}$
  - circle: 2π radians
- Solid angle: $\Omega = \dfrac{A}{r^2}$
  - sphere: 4π steradians





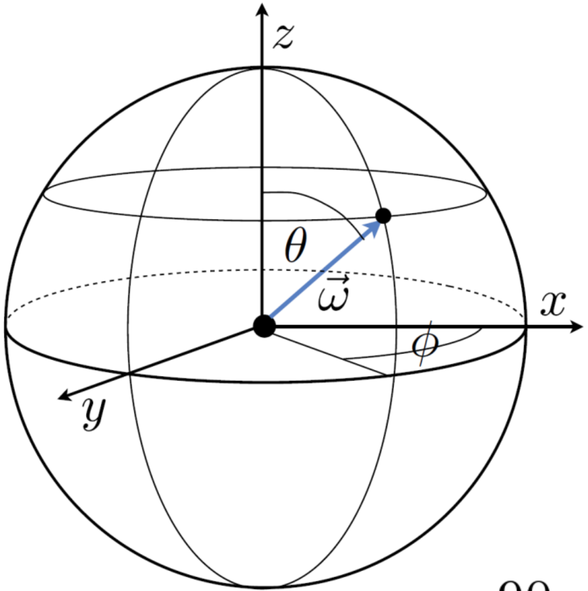
Basic Definitions

- Direction
  - point on the unit sphere
  - parameterized by two angles

$$\vec{\omega} = (\theta, \phi)$$

zenith (θ) azimuth (ϕ)

$$\vec{\omega}_x = \sin\theta\cos\phi$$
$$\vec{\omega}_y = \sin\theta\sin\phi$$
$$\vec{\omega}_z = \cos\theta$$



$$latitude = \frac{90}{\pi}(\pi - \theta) \qquad longitude = \frac{90}{\pi}\phi$$

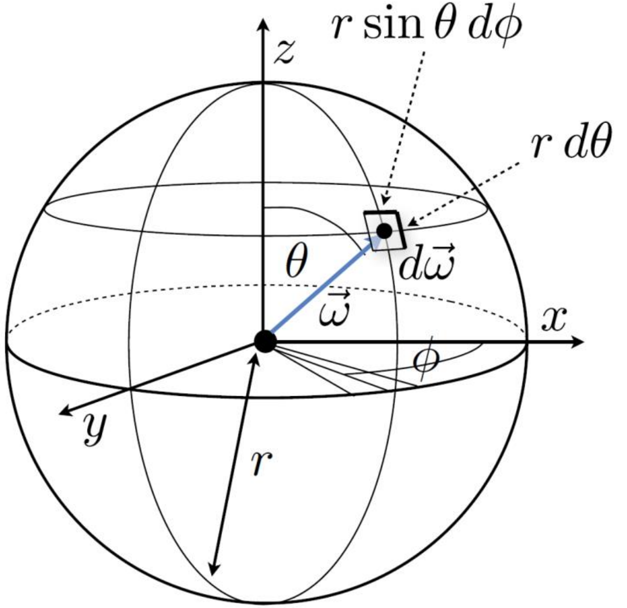
Basic Definitions

- Differential Solid Angle

$$dA = (r d\theta)(r \sin\theta d\phi)$$

$$d\vec{\omega} = \frac{dA}{r^2} = \sin\theta d\theta d\phi$$

$$\Omega = \int_{S^2} d\vec{\omega} = \int_0^{2\pi} \int_0^{\pi} \sin\theta d\theta d\phi = 4\pi$$



Basic Definitions

- Assume light consists of photons with
  - $\mathbf{x}$: Position
  - $\vec{\omega}$: Direction of motion
  - λ : Wavelength
- Each photon has an energy of: $\frac{hc}{\lambda}$
  - $h \approx 6.63 \cdot 10^{-34}\, m^2 \cdot kg/s$: Planck's constant
  - $c = 299{,}792{,}458\, m/s$: speed of light in vacuum
  - Unit of energy, Joule : $[J = kg \cdot m^2/s^2]$

Radiometry

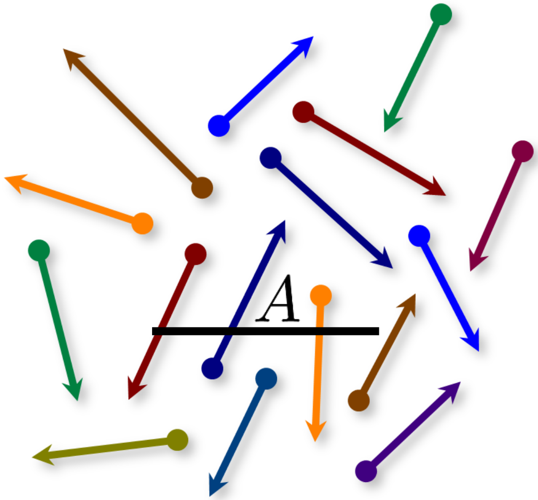
- Basic quantities
  - flux Φ
  - irradiance E
  - radiosity B
  - intensity I
  - radiance L

Radiometry

- Flux (radiant flux, power)
  - total amount of energy passing through surface or space per unit time

$$\Phi(A) \qquad \left[\frac{J}{s} = W\right]$$

  - examples:
    - number of photons hitting a wall per second
    - number of photons leaving a lightbulb per second

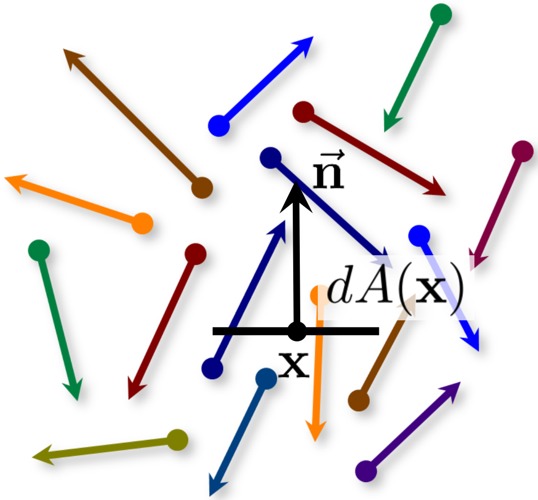


Radiometry

- Irradiance
  - flux per unit area *arriving* at a surface = area density of flux

$$E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})} \left[\frac{W}{m^2}\right]$$

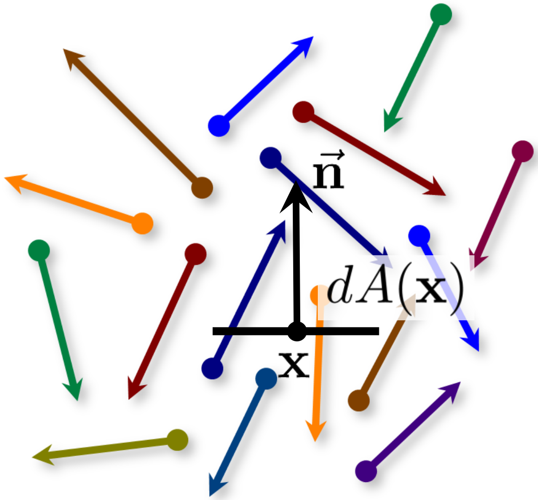
  - example:
    - number of photons hitting a small patch of a wall per second, divided by the size of the patch



Radiometry

- Radiosity
  - flux per unit area *leaving* a surface = area density of flux

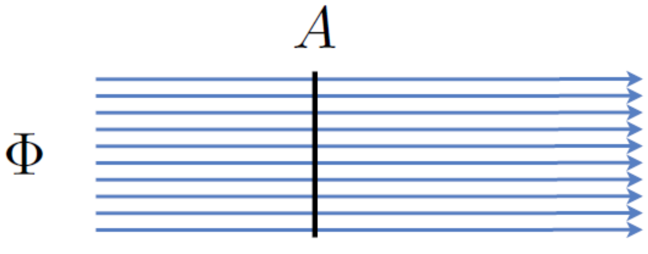
$$B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})} \left[\frac{W}{m^2}\right]$$



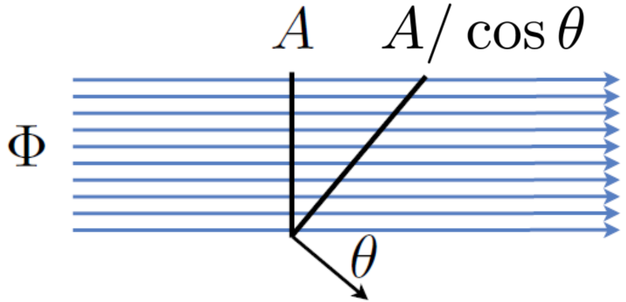
  - example:
    - number of photons reflecting off a small patch of a wall per second, divided by the size of the patch

Radiometry

- Irradiance
  - Lambert's Cosine Law



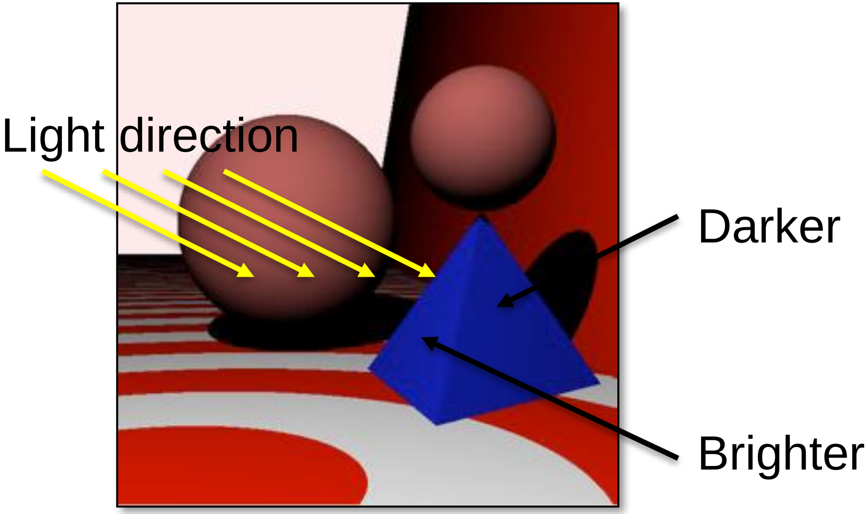
$$E = \frac{\Phi}{A}$$

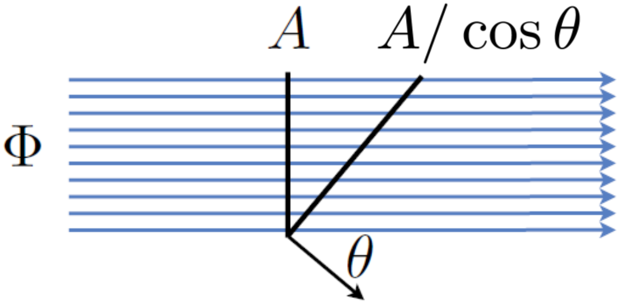


$$E = \frac{\Phi}{A/\cos\theta} = \frac{\Phi}{A}\cos\theta$$

Radiometry

- Irradiance
  - Lambert's Cosine Law



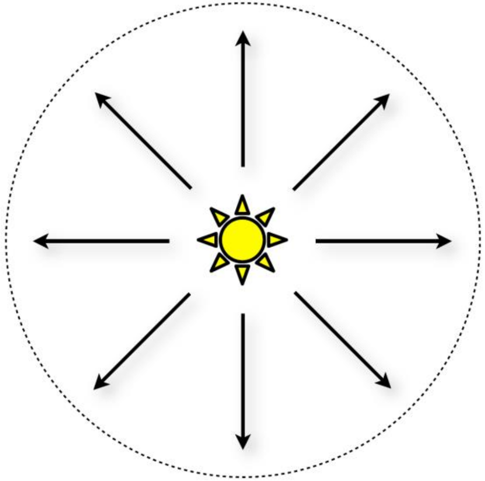


$$E = \frac{\Phi}{A/\cos\theta} = \frac{\Phi}{A}\cos\theta$$

Radiometry

- Radiant intensity
  - Power (flux) per solid angle = directional density of flux

$$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}} \qquad \left[\frac{W}{sr}\right] \qquad \Phi = \int_{S^2} I(\vec{\omega})d\vec{\omega}$$



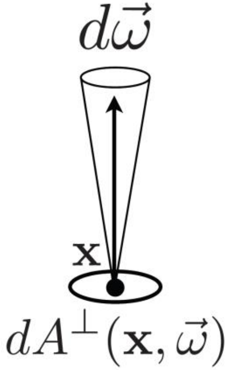
  - example:
    - power per unit solid angle emanating from a point source

$\Phi = 4\pi I$ isotropic point source

Radiometry

- Radiance
  - intensity per unit area = flux density per unit solid angle, per perpendicular unit area

$$L(\mathbf{x},\vec{\omega}) = \frac{dI(\vec{\omega})}{dA(\mathbf{x})} = \frac{d^2\Phi(A)}{d\vec{\omega}\,dA^{\perp}(\mathbf{x},\vec{\omega})} = \frac{d^2\Phi(A)}{d\vec{\omega}\,dA(\mathbf{x})\cos\theta} \qquad \left[\frac{W}{m^2 sr}\right]$$



  - most fundamental for raytracing
  - remains constant along a ray

Radiometry

- Other radiometric quantities can be expressed in terms of radiance
  - Irradiance:

$$L(\mathbf{x},\vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x})d\vec{\omega}} \quad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$L(\mathbf{x},\vec{\omega}) = \frac{dE(\mathbf{x})}{\cos\theta d\vec{\omega}}$$

$$L(\mathbf{x},\vec{\omega})\cos\theta d\vec{\omega} = dE(\mathbf{x})$$

$$\int_{H^2} L(\mathbf{x},\vec{\omega})\cos\theta d\vec{\omega} = E(\mathbf{x})$$

  - Integrate radiance over the hemisphere
  - Same for radiosity

Radiometry

- Other radiometric quantities can be expressed in terms of radiance
  - Flux:

$$E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$\int_A E(\mathbf{x}) dA(\mathbf{x}) = \Phi(A)$$

$$\int_A \int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta d\vec{\omega} dA(\mathbf{x}) = \Phi(A)$$

  - Integrate irradiance over area
  - Integrate radiance over hemisphere and area

Radiometry

- Basic quantities

Flux

$$\Phi(A) \qquad \left[\frac{J}{s} = W\right]$$

Irradiance

$$E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})} \qquad \left[\frac{W}{m^2}\right]$$

Radiosity

$$B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})} \qquad \left[\frac{W}{m^2}\right]$$

Intensity

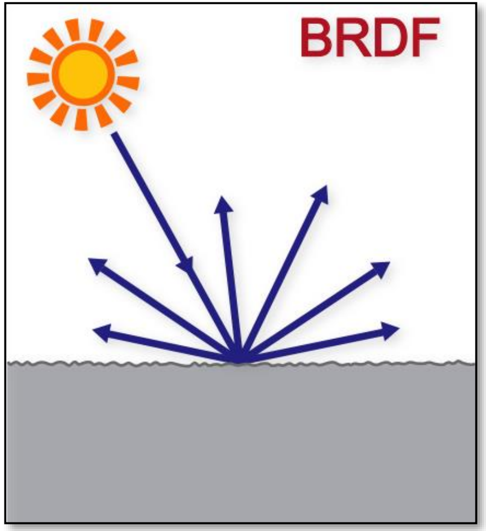
$$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}} \qquad \left[\frac{W}{sr}\right]$$

Radiance

$$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x}) d\vec{\omega}} \qquad \left[\frac{W}{m^2 sr}\right]$$

Reflection Models

- **B**idirectional **R**eflectance **D**istribution **F**unction (BRDF)

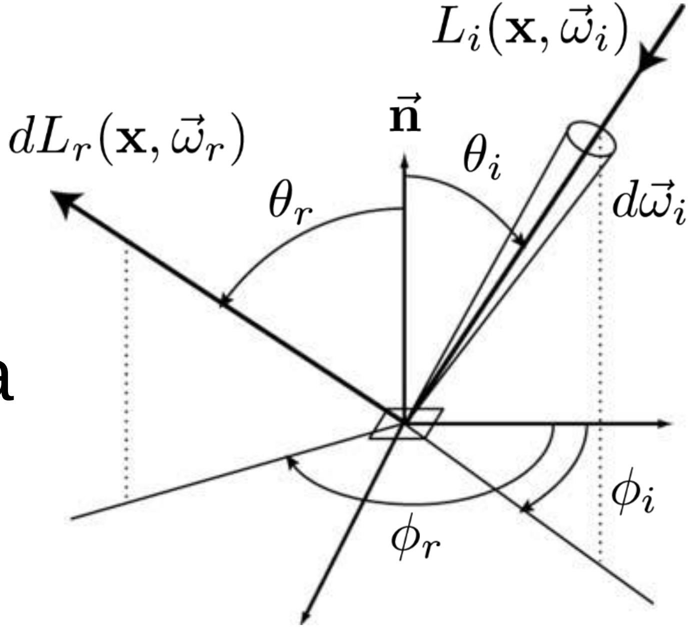


BRDF

- **B**idirectional **R**eflectance **D**istribution **F**unction

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{dE_i(\mathbf{x}, \vec{\omega}_i)}$$

$$= \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i d\vec{\omega}_i} \quad [1/sr]$$



- Differential irradiance due to a cone of directions around $\vec{\omega}_i$

Reflection Equation

- The BRDF provides a relation between incident radiance and differential reflected radiance
- From this we can derive the **Reflection Equation**

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i}$$

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{d\vec{\omega}_i}$$

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i = L_r(\mathbf{x}, \vec{\omega}_r)$$

Reflection Equation

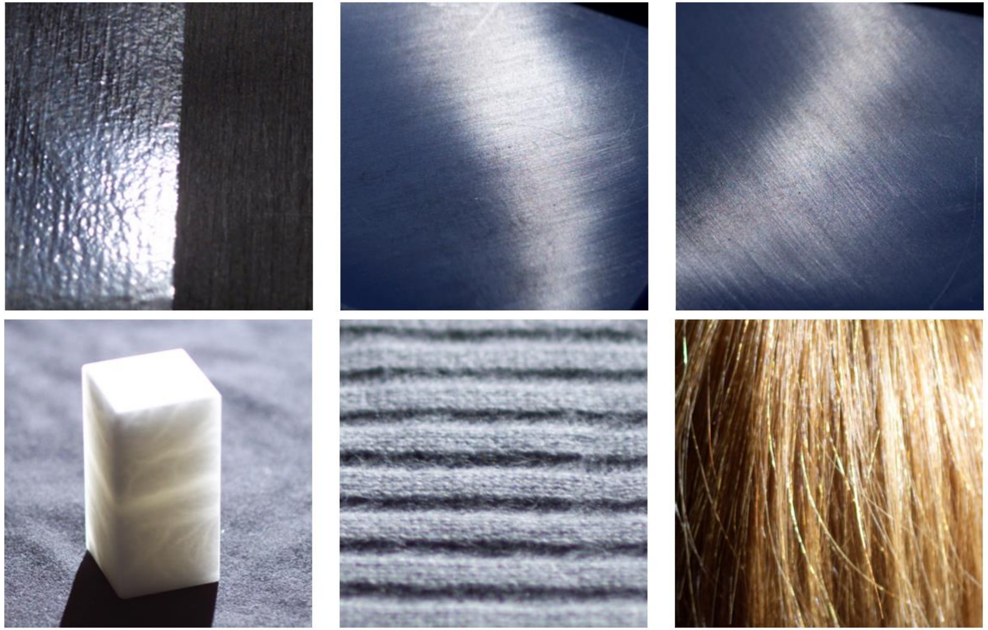
- The Reflection Equation describes a *local illumination* model
  - reflected radiance due to incident illumination from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Complex Reflections

anisotropic reflections

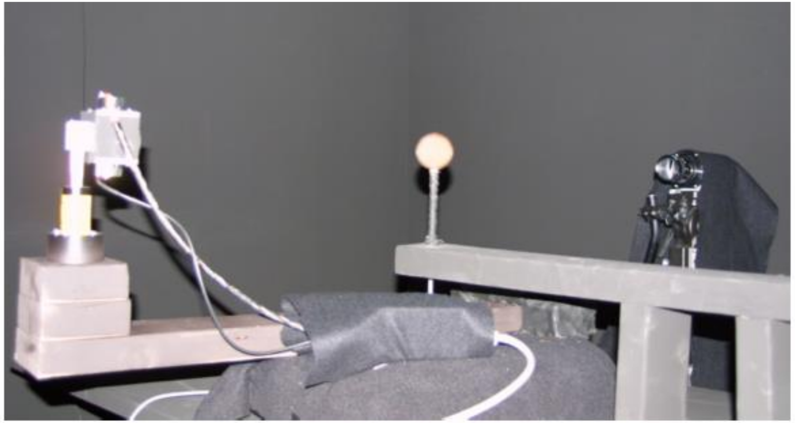
layered materials

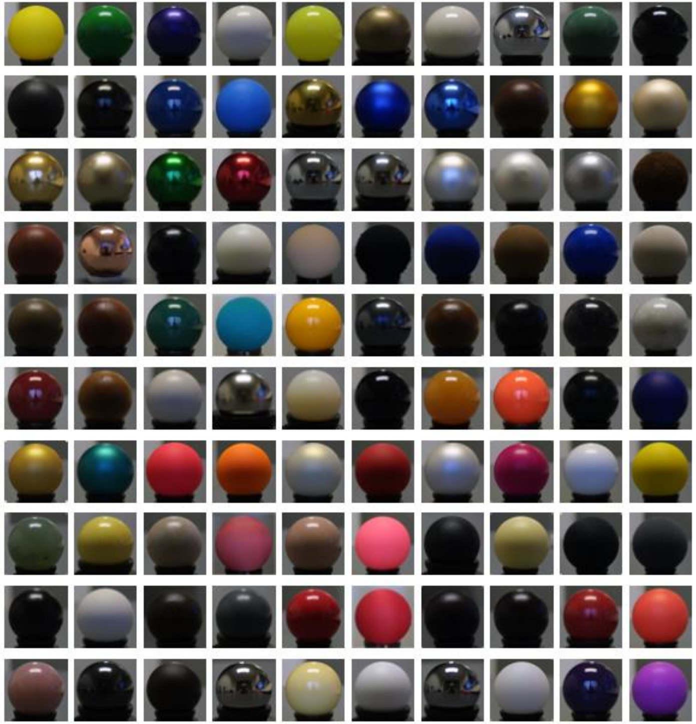


subsurface scattering

volumetric structures

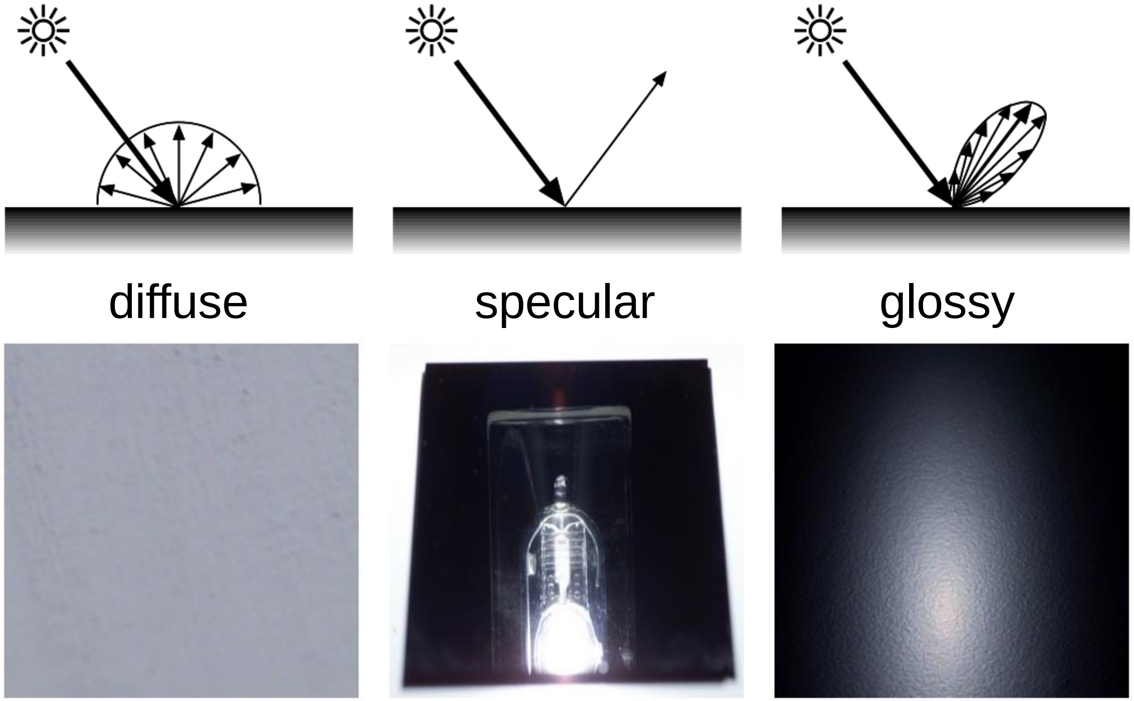
Measuring BRDFs





Matusik et al.: Efficient Isotropic BRDF Measurement, Eurographics Symposium on Rendering 2003

Simpler Reflections



Hendrik Lensch, Efficient Image-Based Appearance Acquisition of Real-World Objects, Ph.D. thesis, 2004

Diffuse Reflection

- For diffuse reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

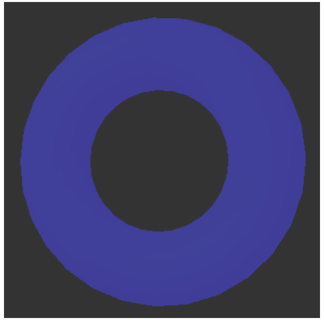
$$L_r(\mathbf{x}) = f_r \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r \, E_i(\mathbf{x})$$

Simple Models

- Exact computation too slow
- OpenGL uses simplified reflection models
- Phong illumination

Ambient Light

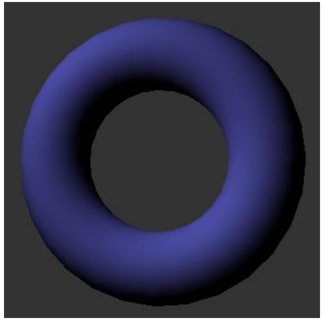


- Scattered by environment
- Coming from all directions
- Reflection independent of
  - Camera position
  - Light position (no light position)
  - Surface orientation
- Reflected intensity: $I = I_a k_a$

light source

material parameter

Diffuse Reflection

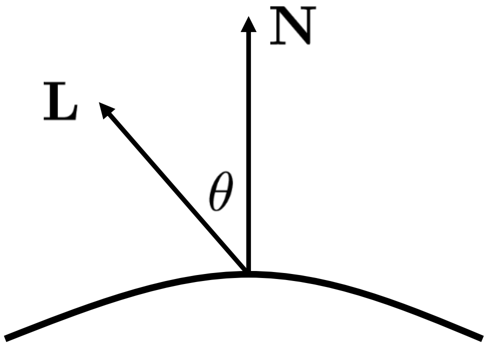


- Directed light I_p
- Reflection dependent on
  - orientation of surface
  - light source position
- Independent of
  - camera position (reflected equally in all directions)
- Reflected intensity:

$$I = I_p k_d \cos\theta$$

$$I = I_p k_d (\mathbf{N} \cdot \mathbf{L})$$

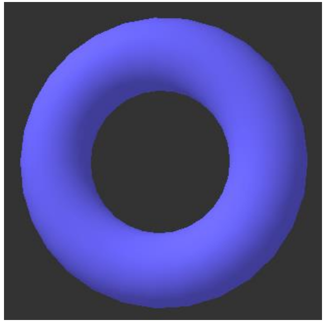
ve dot product tor



A Simple Model

- Sum up ambient light and diffuse reflection:

$$I = I_a k_a + I_p k_d (\mathbf{N} \cdot \mathbf{L})$$



Attenuation

- Quadratic attenuation due to spatial radiation

$$f_{att} = \frac{1}{d_L^2}$$

- A model often used in Graphics (OpenGL)

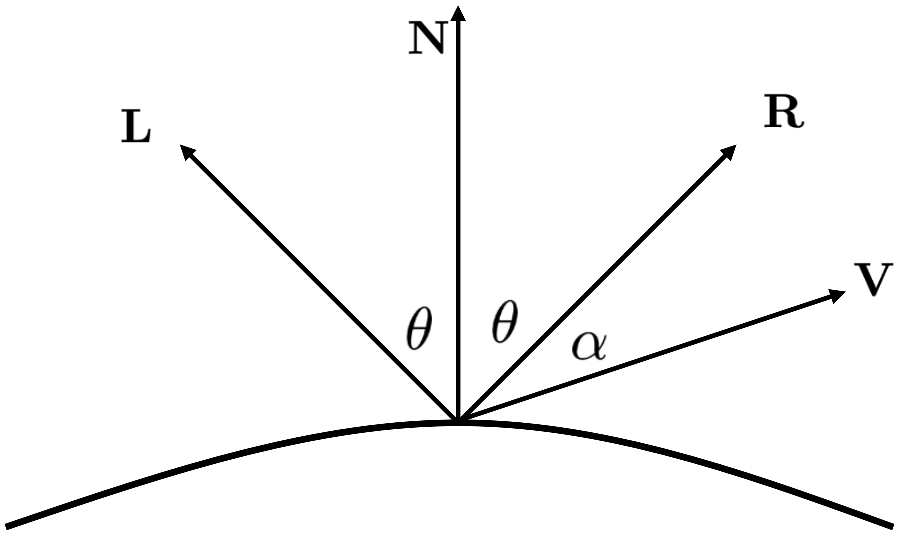
$$f_{att} = \min\left(\frac{1}{c_1 + c_2 d_L + c_3 d_L^2}, 1\right)$$

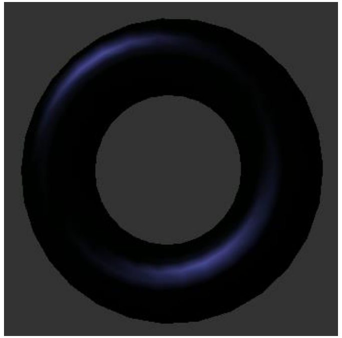
- Include attenuation

$$I = I_a k_a + f_{att} I_p k_d (\mathbf{N} \cdot \mathbf{L})$$

Specular Reflection

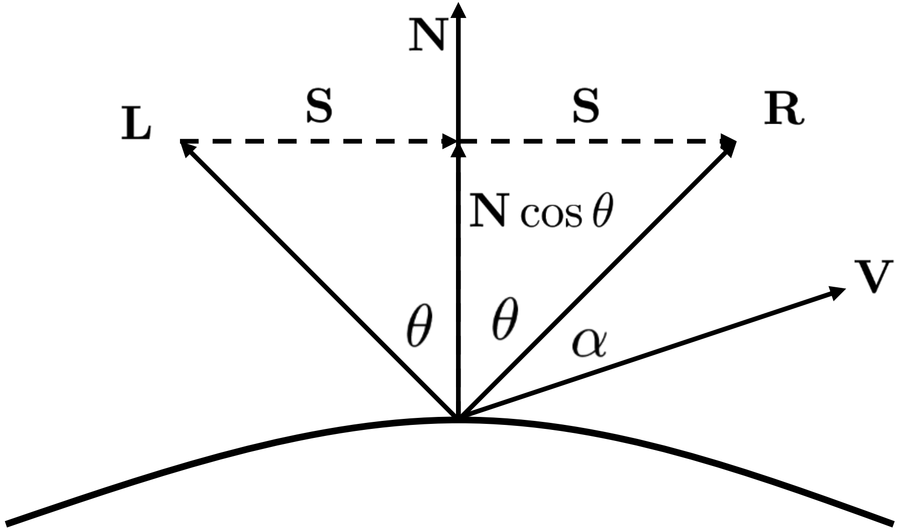
- Depends on the angle between the reflection and viewing ray

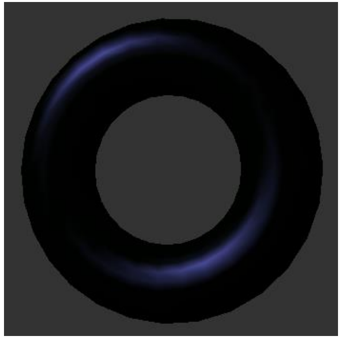




Specular Reflection

- Compute by simple linear algebra



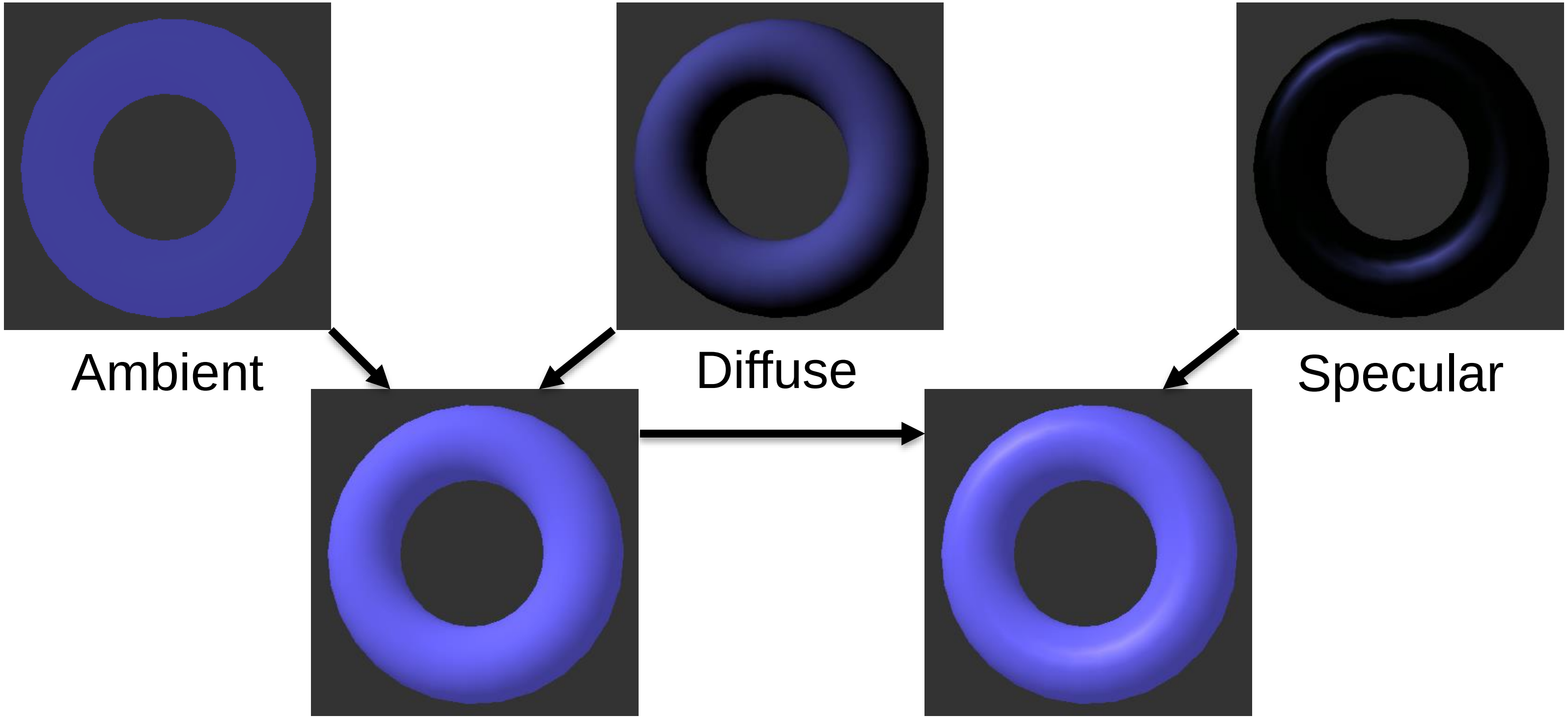


$$\mathbf{R} = \mathbf{N}\cos\theta + \mathbf{S}$$

$$\mathbf{R} = 2\mathbf{N}\cos\theta - \mathbf{L} = 2\mathbf{N}(\mathbf{N}\cdot\mathbf{L}) - \mathbf{L}$$

$$\cos\alpha = \mathbf{R}\cdot\mathbf{V} = (2\mathbf{N}(\mathbf{N}\cdot\mathbf{L}) - \mathbf{L})\cdot\mathbf{V}$$

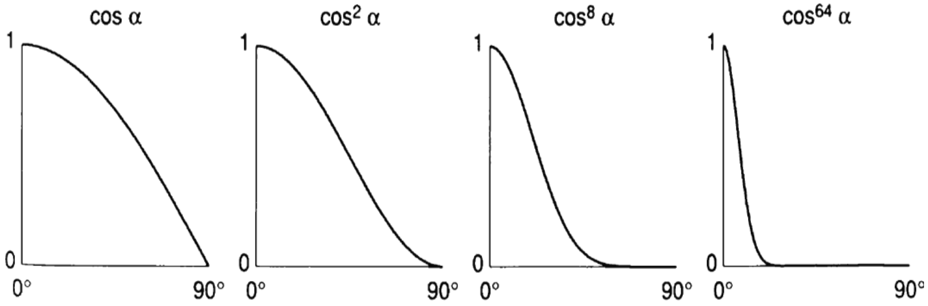
Ambient + Diffuse + Specular



Phong Illumination Model

- Approximates specular reflection by cosine powers

$$I_\lambda = I_{a_\lambda} k_a O_{d_\lambda} + f_{att} I_{p_\lambda} \left[k_d O_{d_\lambda} (\mathbf{N} \cdot \mathbf{L}) + k_s (\mathbf{R} \cdot \mathbf{V})^n \right]$$



Extensions

- Specular colors

$$I_\lambda = I_{a_\lambda} k_a + f_{att} I_{p_\lambda} \left[k_d (\mathbf{N} \cdot \mathbf{L}) + k_s (\mathbf{R} \cdot \mathbf{V})^n \right]$$

Material dependent constants

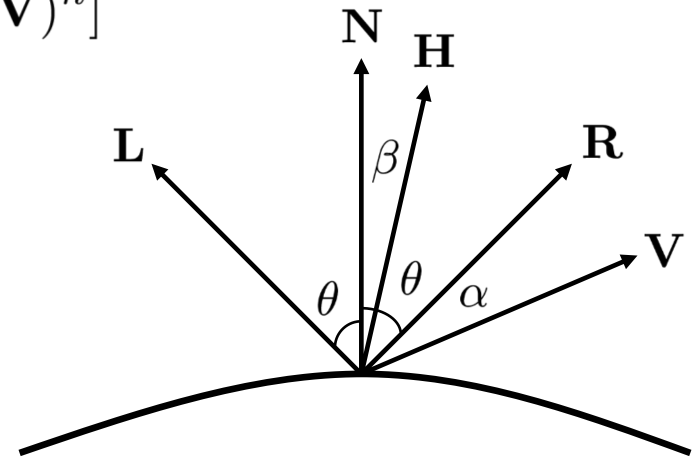
Extensions

- Specular colors

$$I_\lambda = I_{a_\lambda} k_a + f_{att} I_{p\lambda} \left[k_d (\mathbf{N} \cdot \mathbf{L}) + k_s (\mathbf{R} \cdot \mathbf{V})^n \right]$$

- Halfway vector (faster)

$$\cos^n \beta = (\mathbf{N} \cdot \mathbf{H})^n \qquad \mathbf{H} = \frac{\mathbf{L} + \mathbf{V}}{\|\mathbf{L} + \mathbf{V}\|}$$



- Multiple light sources

$$I_\lambda = I_{a_\lambda} k_a + \sum_{1 \le i \le m} f_{att_i} I_{p_{\lambda_i}} \left[k_d (\mathbf{N} \cdot \mathbf{L}_i) + k_s (\mathbf{R}_i \cdot \mathbf{V})^n \right]$$

End