Visual Computing:

Image Segmentation

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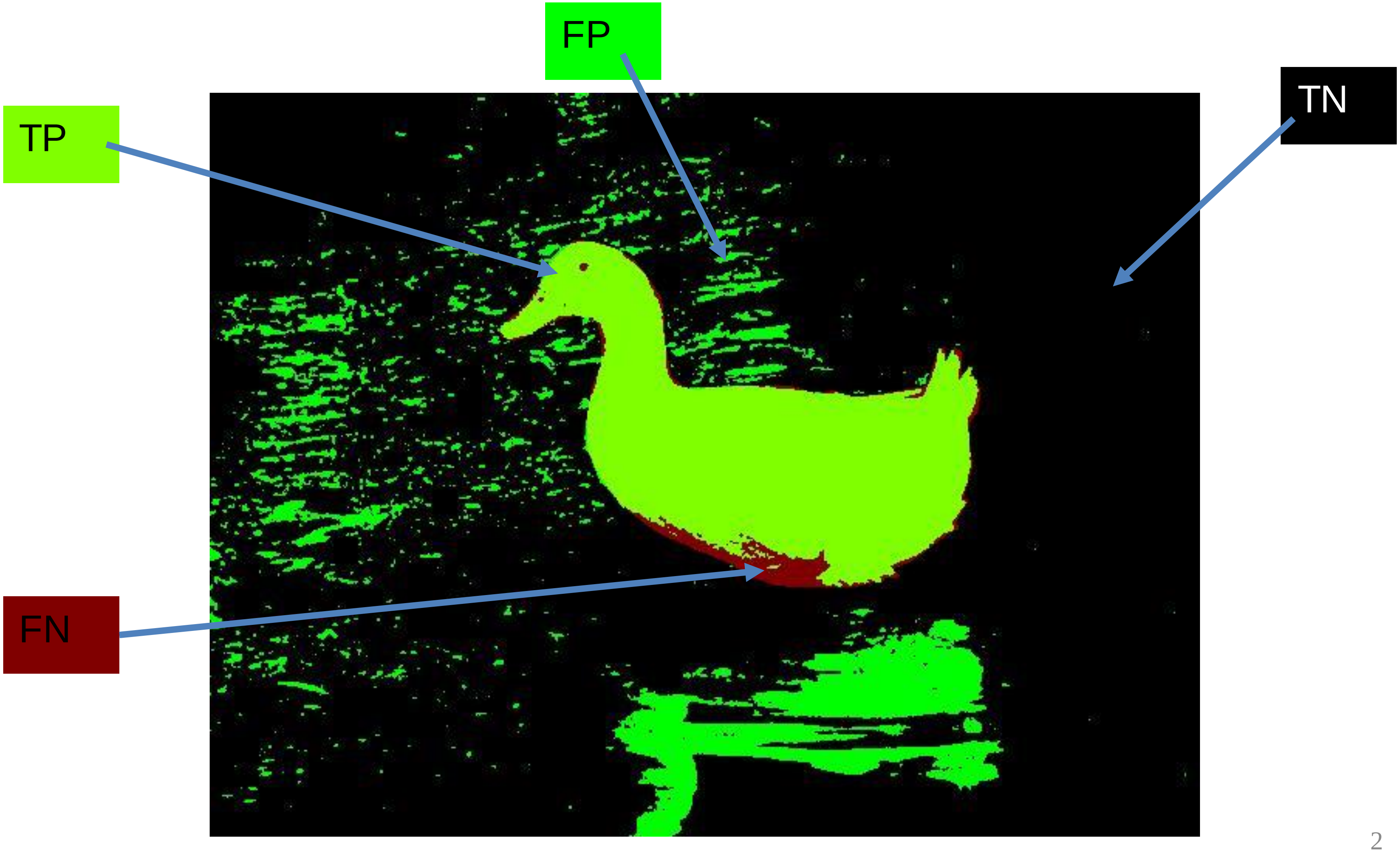
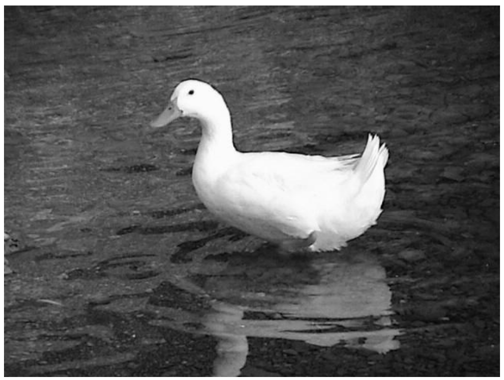
ETH

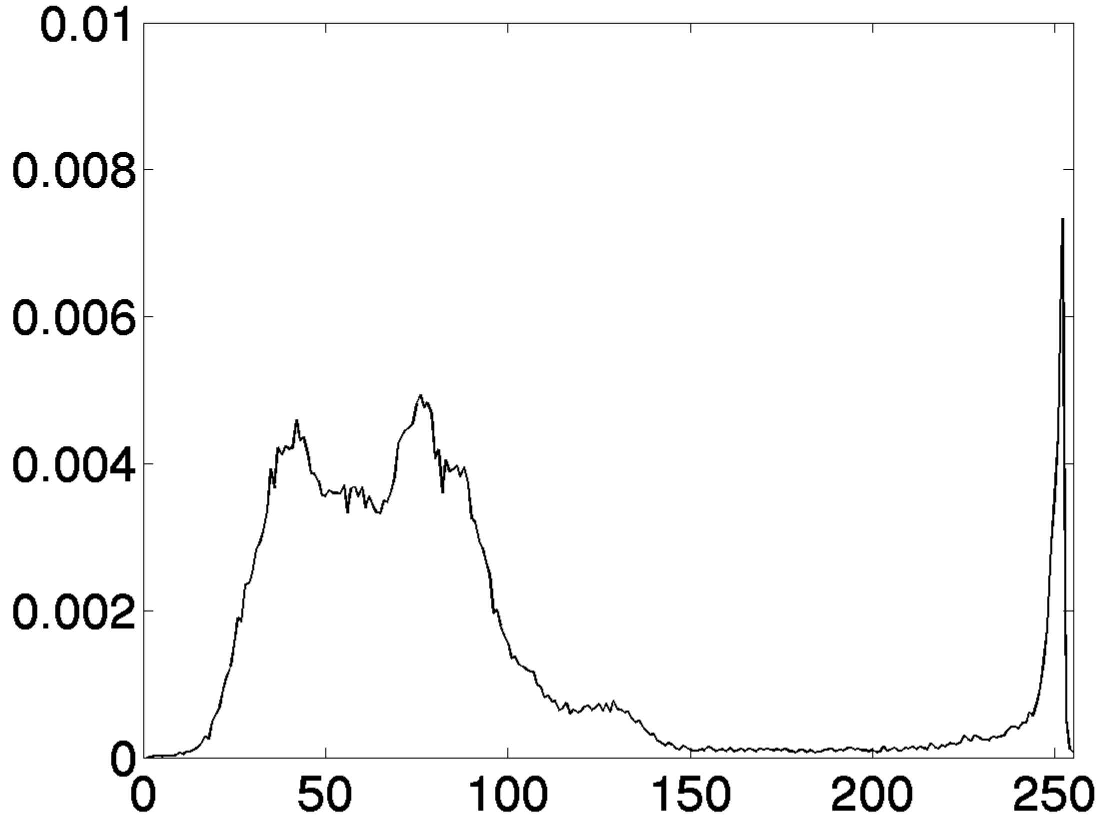
inf | Informatik
Computer Science

Classification outcomes

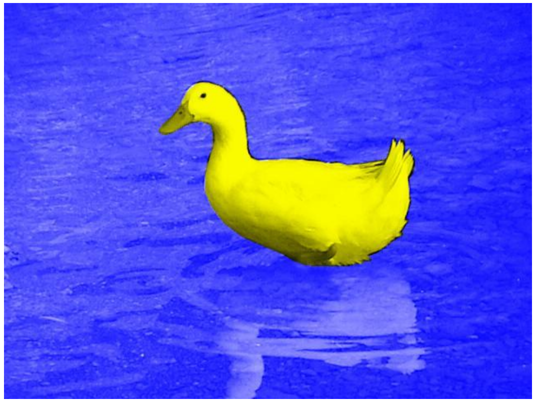


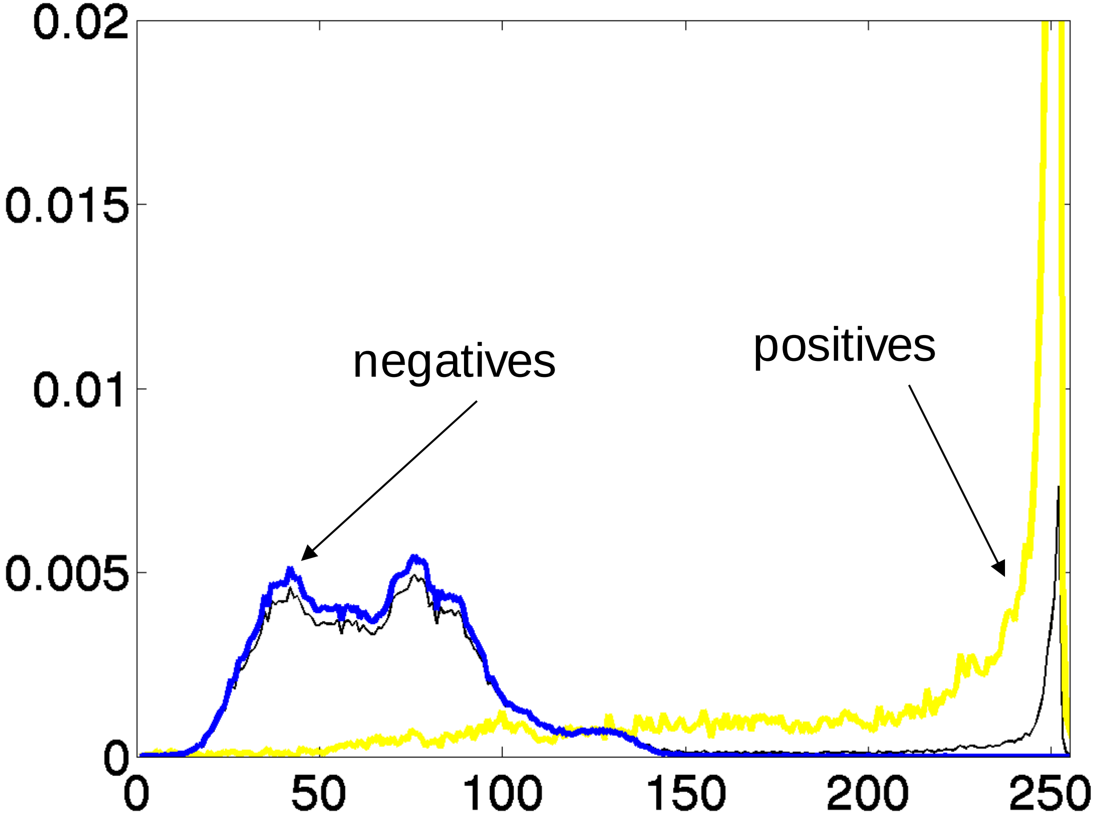
Greylevel Histograms





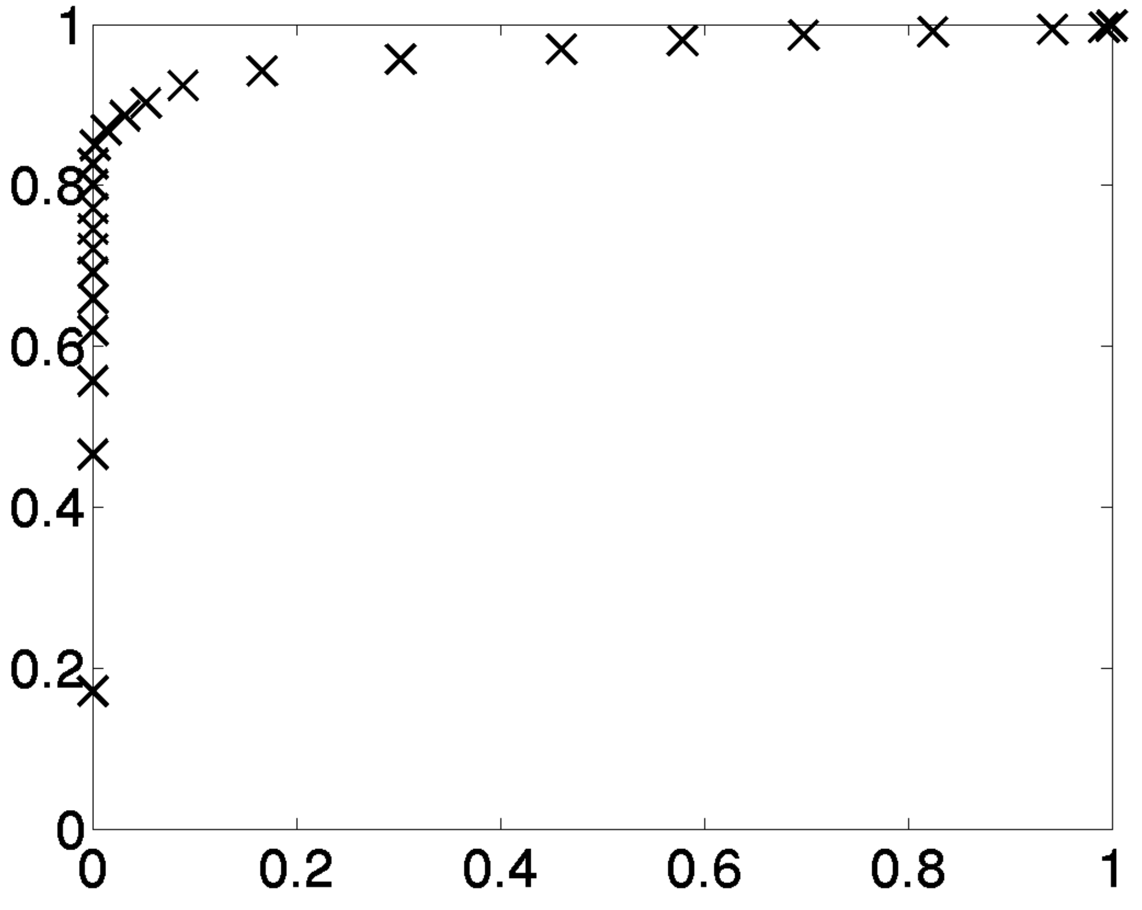
Positives and Negatives





ROC curve

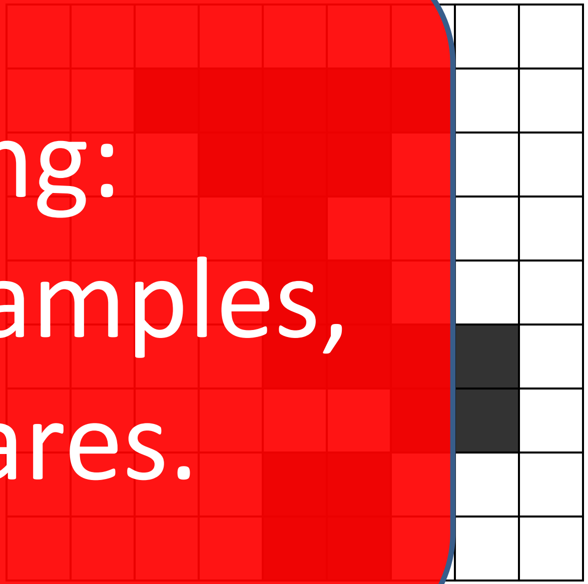
TP fraction



FP fraction

Pixel connectivity

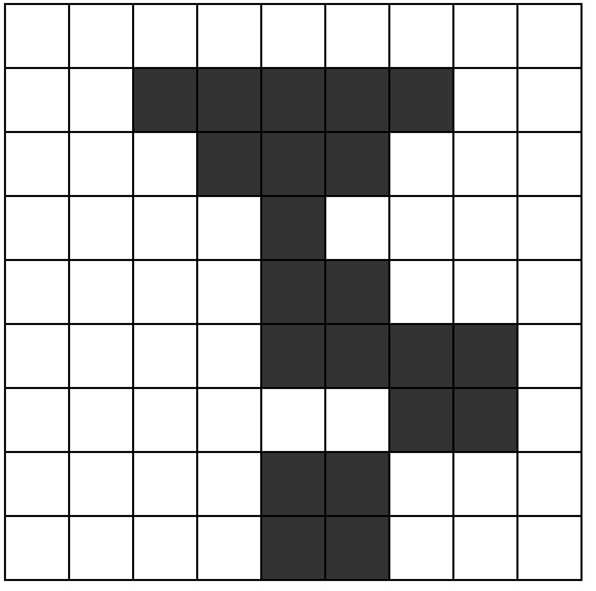
- We need to define which pixels are neighbors.
- Are the dark pixels in this array connected?



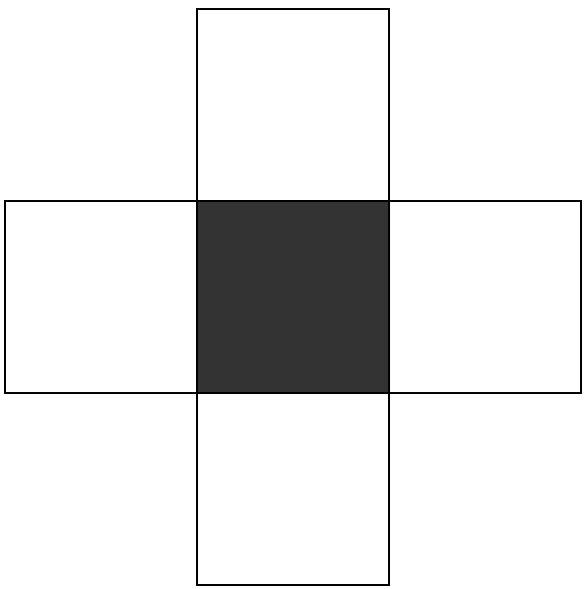
Warning:
Pixels are samples, not squares.

Pixel connectivity

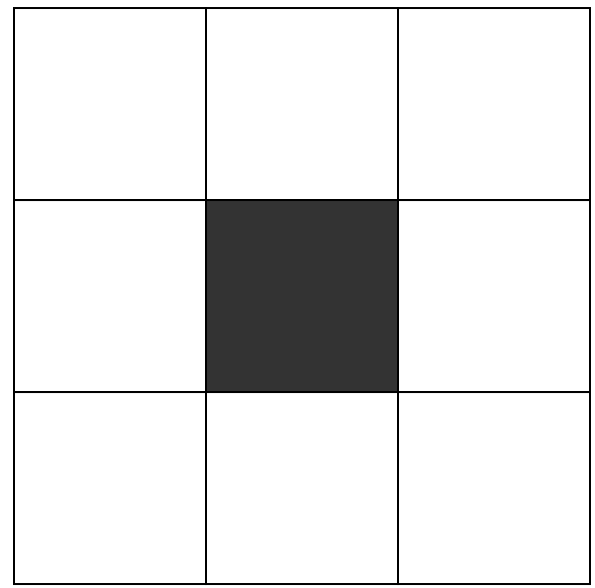
- We need to define which pixels are neighbors.
- Are the dark pixels in this array connected?



Pixel Neighborhoods



4-neighborhood



8-neighborhood

Pixel paths

- A 4-connected path between pixels p_1 and p_n is a set of pixels $\{p_1, p_2, \ldots, p_n\}$ such that p_i is a 4-neighbor of p_{i+1}, $i=1,\ldots,n-1$.

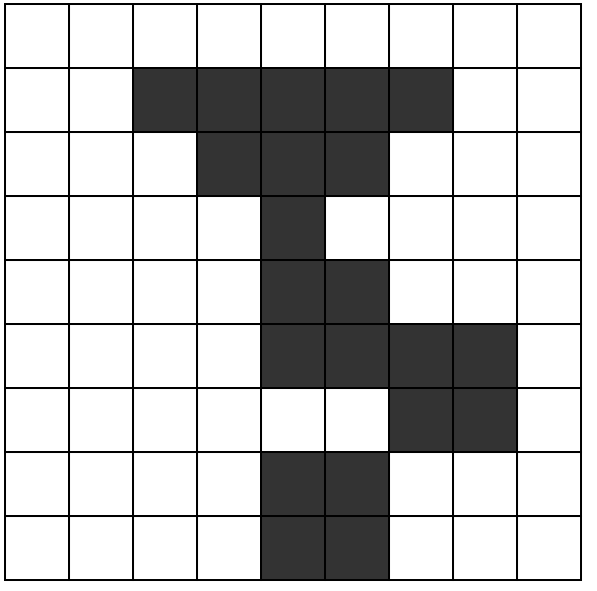
- In an 8-connected path, p_i is an 8-neighbor of p_{i+1}.

Connected regions

- A region is 4-connected if it contains a 4-connected path between any two of its pixels.
- A region is 8-connected if it contains an 8-connected path between any two of its pixels.

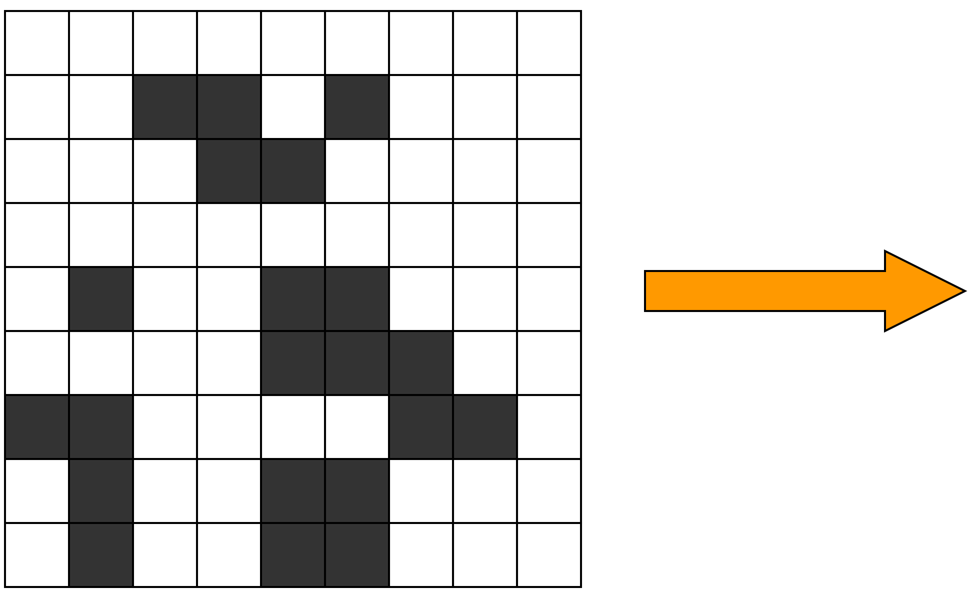
Connected regions

- Now what can we say about the dark pixels in this array?
- What about the light pixels?



Connected components labelling

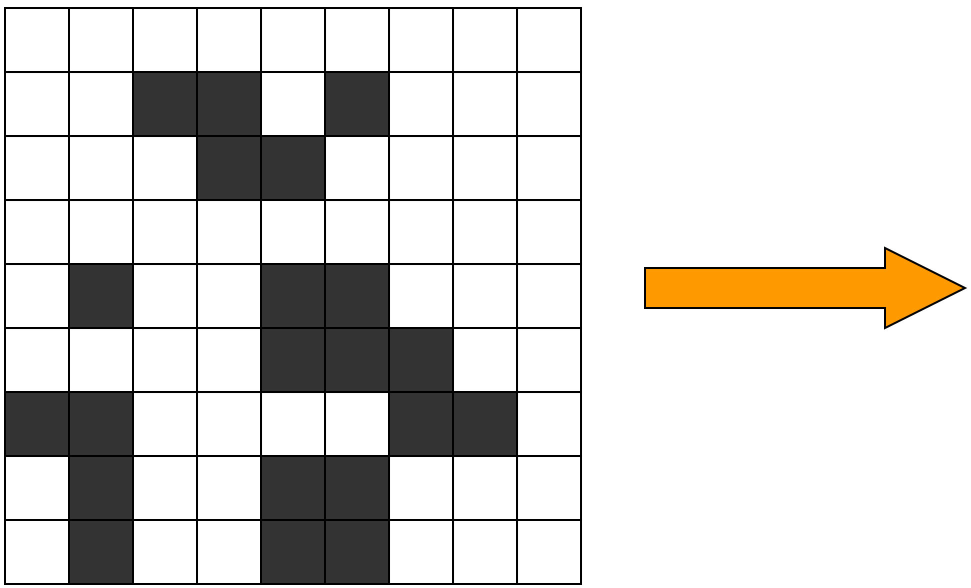
- Labels each connected component of a binary image with a separate number.



1	1	1	1	1	1	1	1	1
1	1	2	2	1	3	1	1	1
1	1	1	2	2	1	1	1	1
1	1	1	1	1	1	1	1	1
1	4	1	1	5	5	1	1	1
1	1	1	1	5	5	5	1	1
6	6	1	1	1	1	5	5	1
7	6	1	1	8	8	1	1	1
7	6	1	1	8	8	1	1	1

Foreground labelling

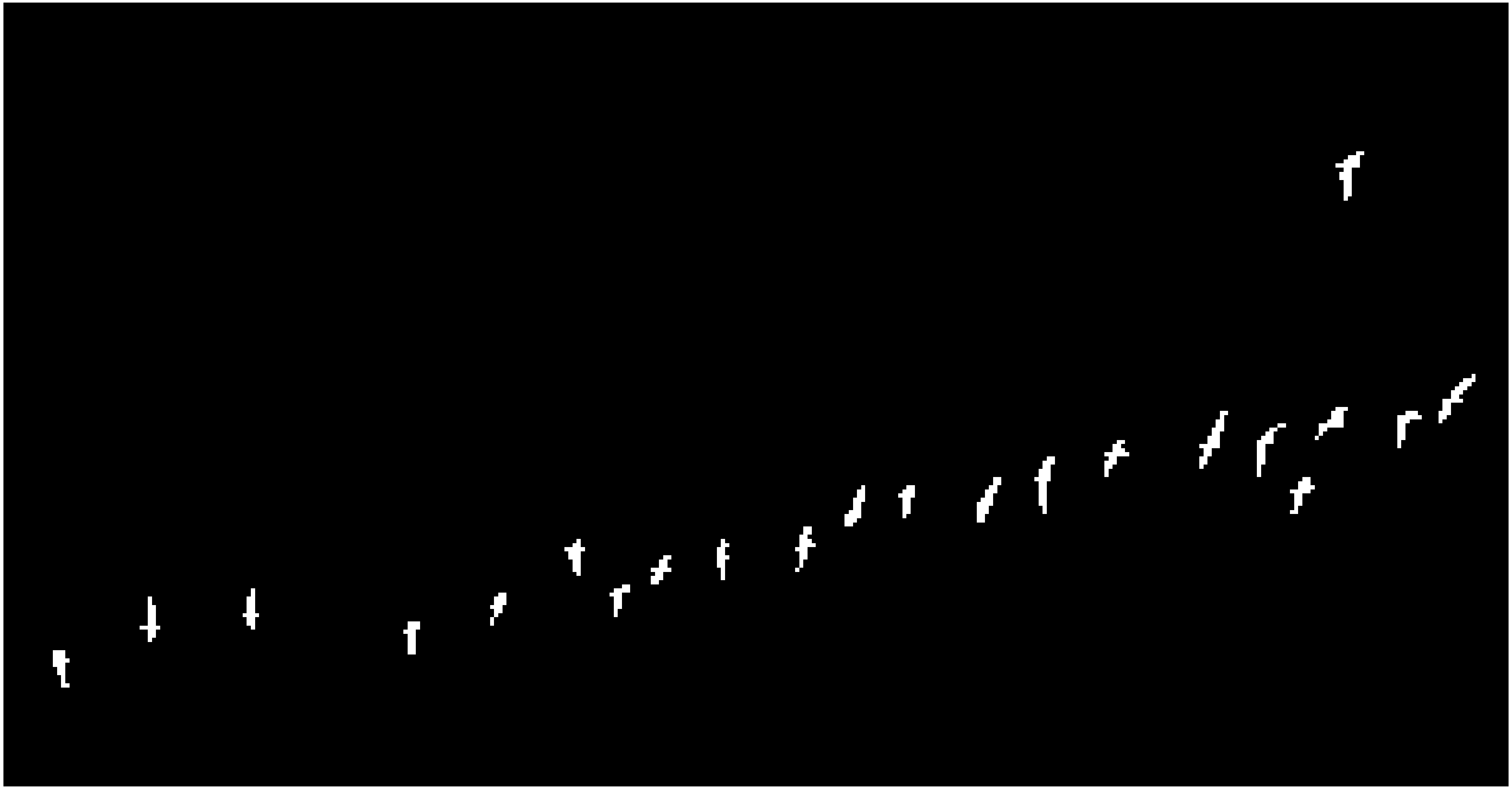
- Only extract the connected components of the foreground



1	1	1	1	1	1	1	1	1
1	1	0	0	1	0	1	1	1
1	1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1	1
1	0	1	1	0	0	1	1	1
1	1	1	1	0	0	0	1	1
0	0	1	1	1	1	0	0	1
2	0	1	1	0	0	1	1	1
2	0	1	1	0	0	1	1	1

Goose detector

Goose detector



Region Growing

- Start from a seed point or region.
- Add neighboring pixels that satisfy the criteria defining a region.
- Repeat until we can include no more pixels.

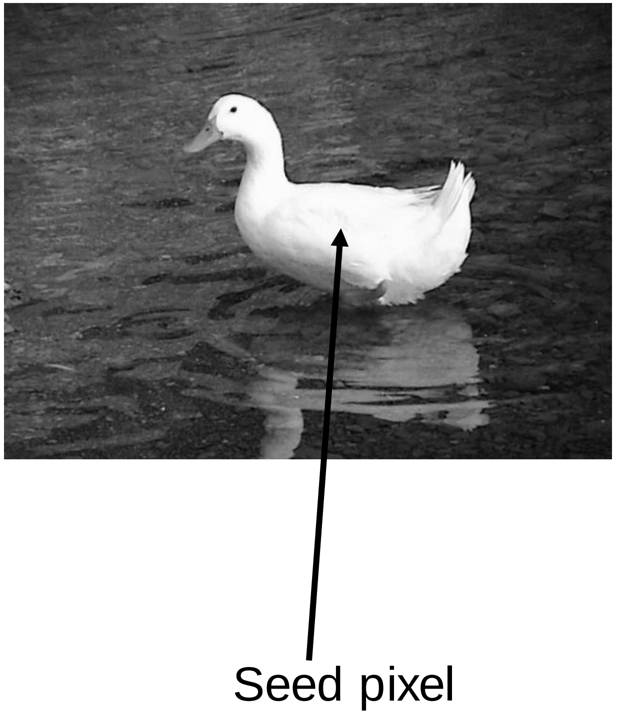
Region Growing

```
def regionGrow(I, seed):
   X, Y = I.shape
   visited = np.zeros((X,Y))
   visited[seed] = 1
   boundary = []
   boundary.append(seed)
   while len(boundary) > 0:
      nextPoint = boundary.pop()
      if include(nextPoint, seed):
            visited[nextPoint] = 2
            for (x, y) in neighbors(nextPoint):
                  if visited[x,y] == 0:
                        boundary.append((x, y))
                        visited[x,y] = 1
```

Region Growing example

- Pick a single seed pixel
- Inclusion test is up to you:

```
def include(p, seed):
   test = ??
   return test
```




T=150

Variations

- Seed selection
- Inclusion criteria
- Boundary constraints and snakes

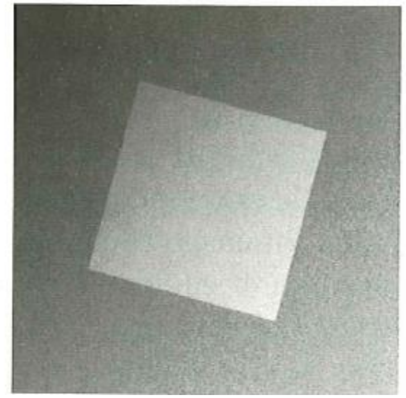
Seed selection

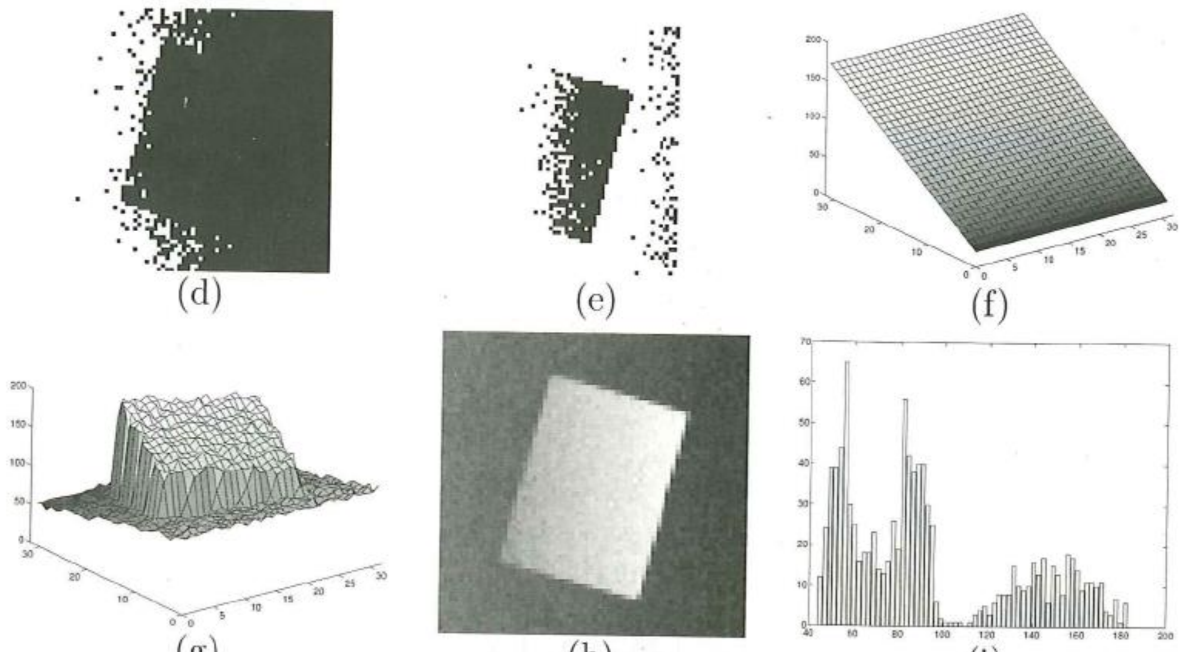
- Point and click seed point
- Seed region
    - By hand
    - Automatically, e.g., from a conservative thresholding.
- Multiple seeds
    - Automatically labels the regions

Inclusion criteria

- Greylevel thresholding
- Greylevel distribution model
  - Use mean μ and standard deviation σ in seed region:
  - Include if $(I(x, y) - \mu)^2 < (n\sigma)^2$. Eg: $n = 3$.
  - Can update the mean and standard deviation after every iteration.
- Color or texture information

Inclusion criteria ?





(d) (e) (f)

(g) (h) (i)

e.g. $(a.I+b.x+c.y+d)^2$<threshold
(keep refitting a,b,c to included pts)

(j)

Snakes

- A snake is an *active contour*
- It is a polygon, i.e., an ordered set of points joined up by lines
- Each point on the contour moves away from the seed while its image neighborhood satisfies an inclusion criterion
- Often the contour has smoothness constraints

Snakes

- The algorithm iteratively minimizes an energy function:

- $E = E_{tension} + E_{stiffness} + E_{image}$

- See Kass, Witkin, Terzopoulos, IJCV 1988

Example

Interim Summary

- Segmentation is hard
- But it is easier if you define the task carefully
    - Is the segmentation task binary or continuous?
    - What are the regions of interest?
    - How accurately must the algorithm locate the region boundaries?
- Research problems remain!

Foreground-Background segmentation

Roundabout example

- Input

- Output

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Distance Measures

Plain Background-subtraction metric:

$$\mathbf{I}_{\alpha} = \left| \mathbf{I} - \mathbf{I}_{bg} \right| > \mathbf{T}$$

$\mathbf{T}$ = [20 20 10] (for example)

$\mathbf{I_{bg}}$ = Background Image

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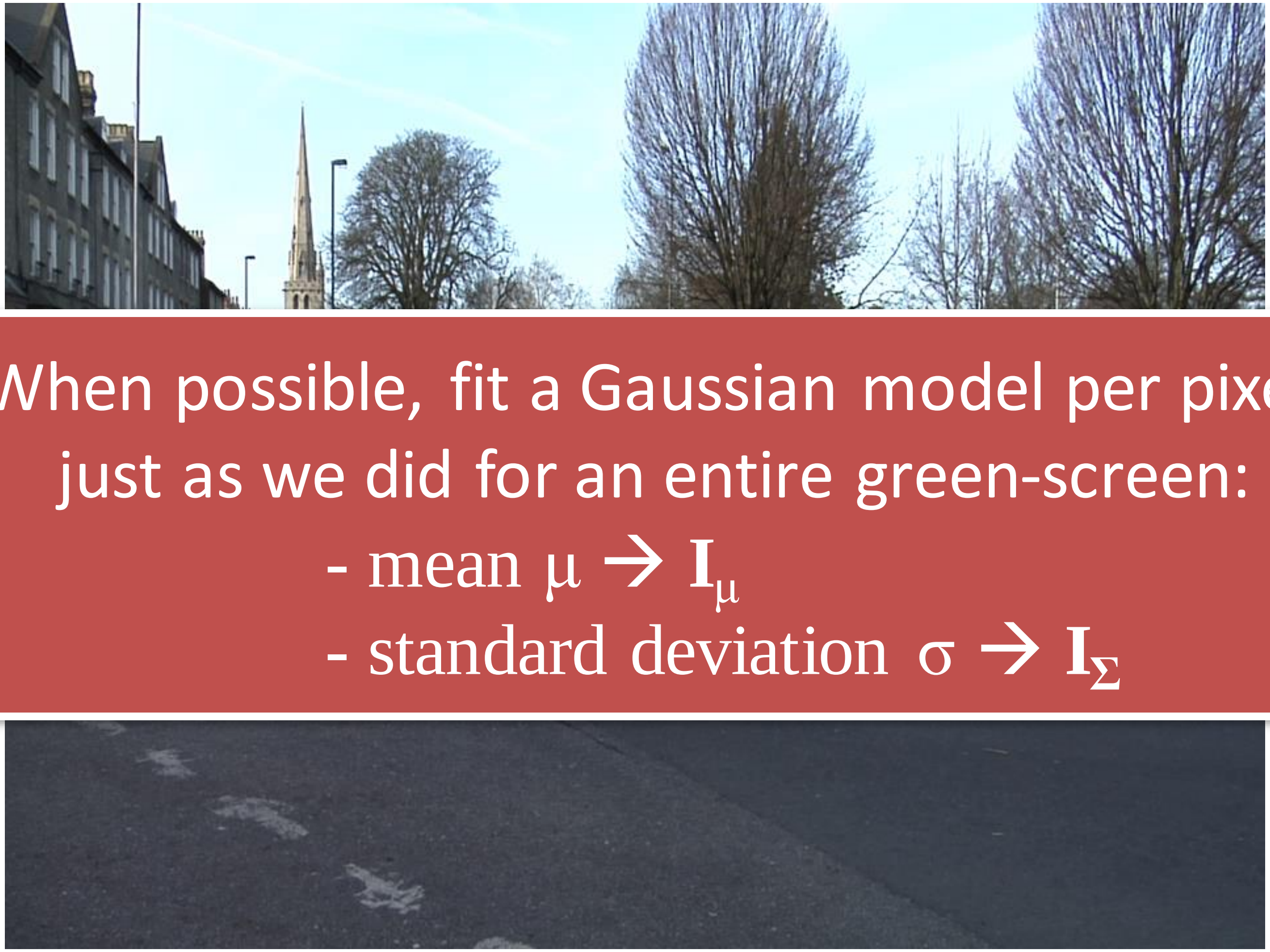
Where Does I_{bg} Come From?



When possible, fit a Gaussian model per pixel, just as we did for an entire green-screen:

- mean $\mu \rightarrow I_\mu$
- standard deviation $\sigma \rightarrow I_\Sigma$

Note: Outdoor backgrounds change over time!

Distance Measures

Plain Background-subtraction metric:

$$\mathbf{I}_{\alpha} = \left|\mathbf{I} - \mathbf{I}_{bg}\right| > \mathbf{T}$$

$\mathbf{T}$ = [20 20 10] (for example)

$\mathbf{I_{bg}}$ = Background Image

or better

$$\mathbf{I}_{\alpha} = \sqrt{\left(\mathbf{I} - \mathbf{I}_{bg}\right)^{T} \Sigma^{-1} \left(\mathbf{I} - \mathbf{I}_{bg}\right)} > \mathbf{T} = 4 \quad \text{(for example)}$$

Σ background pixel appearance covariance matrix (computed separately for each pixel, from many examples) (sometimes need more than one Gaussian, use Gaussian Mixture Models)

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A Word About Shadows

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A Word About Shadows

What happened
to the color here?

Gaussian's symmetry
could mislead a little…
(Brighter-only example)

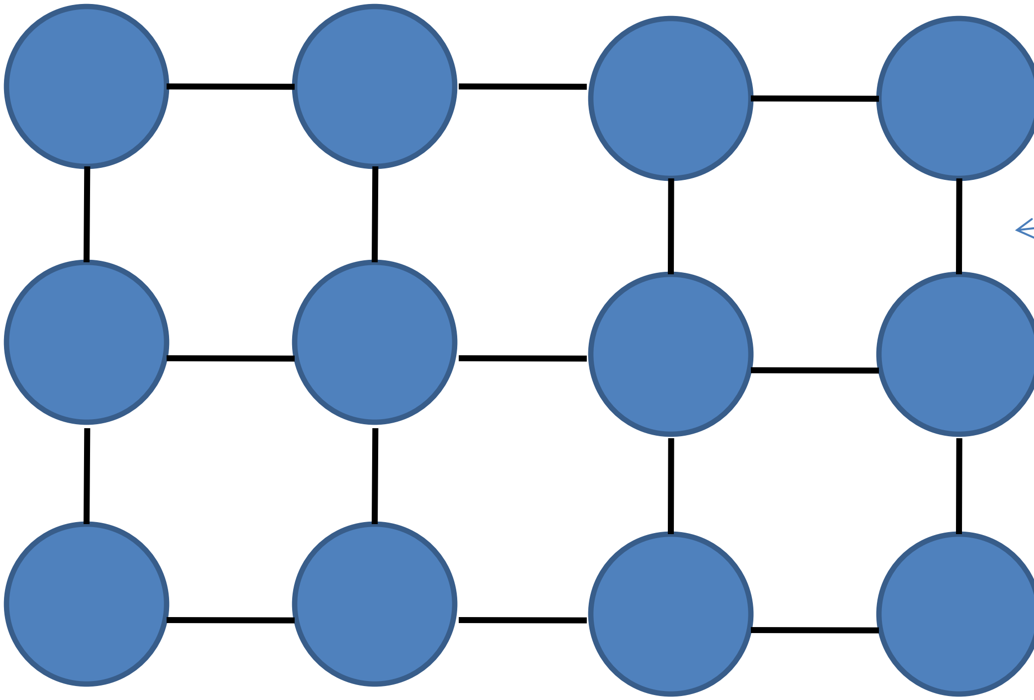
Adding spatial relations

Markov Random Fields

- Markov chains have 1D structure
  - At every time, there is one state.
  - This enabled use of dynamic programming.
- Markov Random Fields break this 1D structure.
  - Field of sites, each of which has a label, simultaneously.
  - Label at one site dependent on others, no 1D structure to dependencies.
  - This means no optimal, efficient algorithms, except for 2-label problems.

Adapted from Derek Hoiem

Markov Random Fields



Node y_i: pixel label

Edge: constrained pairs

Cost to assign a label to each pixel

Cost to assign a pair of labels to connected pixels

$$Energy(\mathbf{y};\theta,data)=\sum_i \psi_1(y_i;\theta,data)+\sum_{i,j\in edges}\psi_2(y_i,y_j;\theta,data)$$

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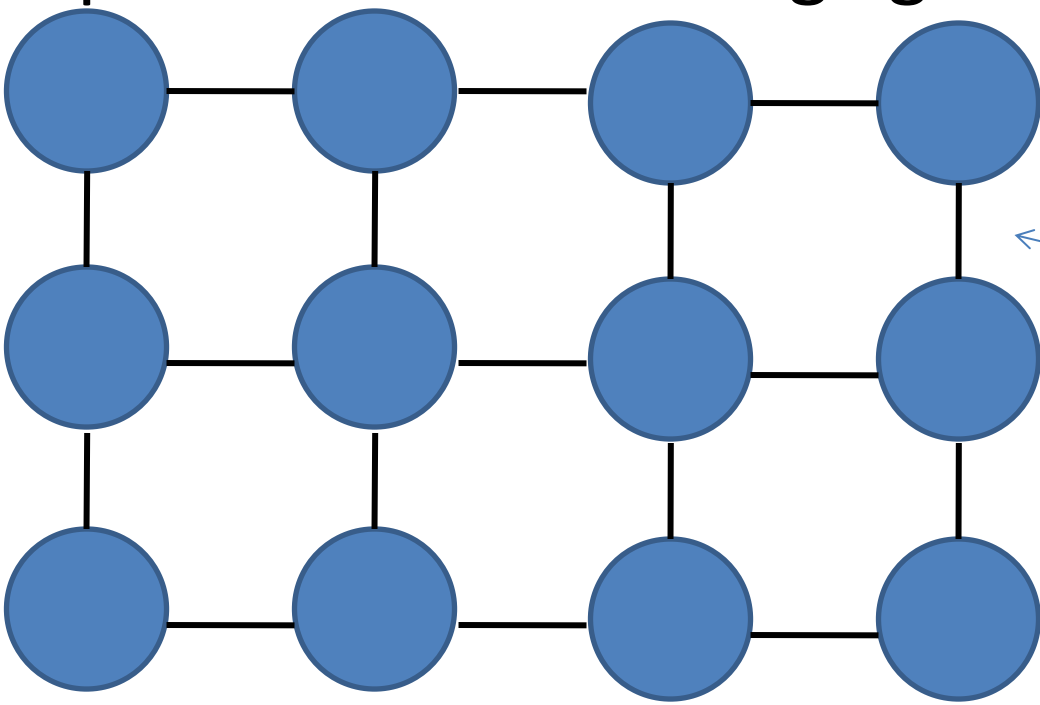
Slides from Derek Hoiem

Markov Random Fields

- Example: “label smoothing” grid



Unary potential

0: -logP(y_i = 0 ; data)
1: -logP(y_i = 1 ; data)

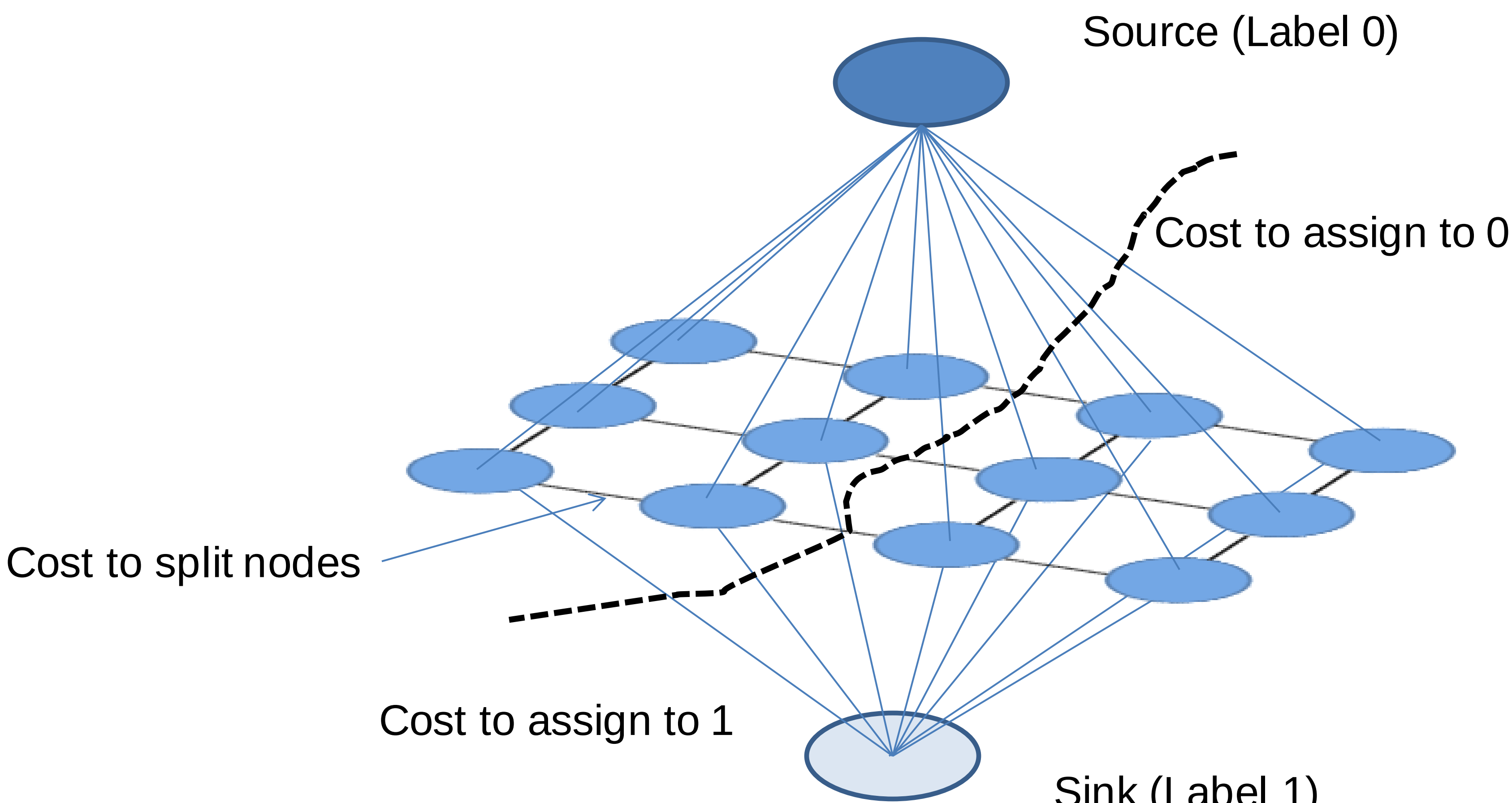
Pairwise Potential

	0	1
0	0	K
1	K	0

$$Energy(\mathbf{y};\theta,data) = \sum_i \psi_1(y_i;\theta,data) + \sum_{i,j\in edges} \psi_2(y_i,y_j;\theta,data)$$

Slides from Derek Hoiem

Solving MRFs with graph cuts



$$Energy(\mathbf{y};\theta,data)=\sum_i \psi_1(y_i;\theta,data)+\sum_{i,j\in edges}\psi_2(y_i,y_j;\theta,data)$$

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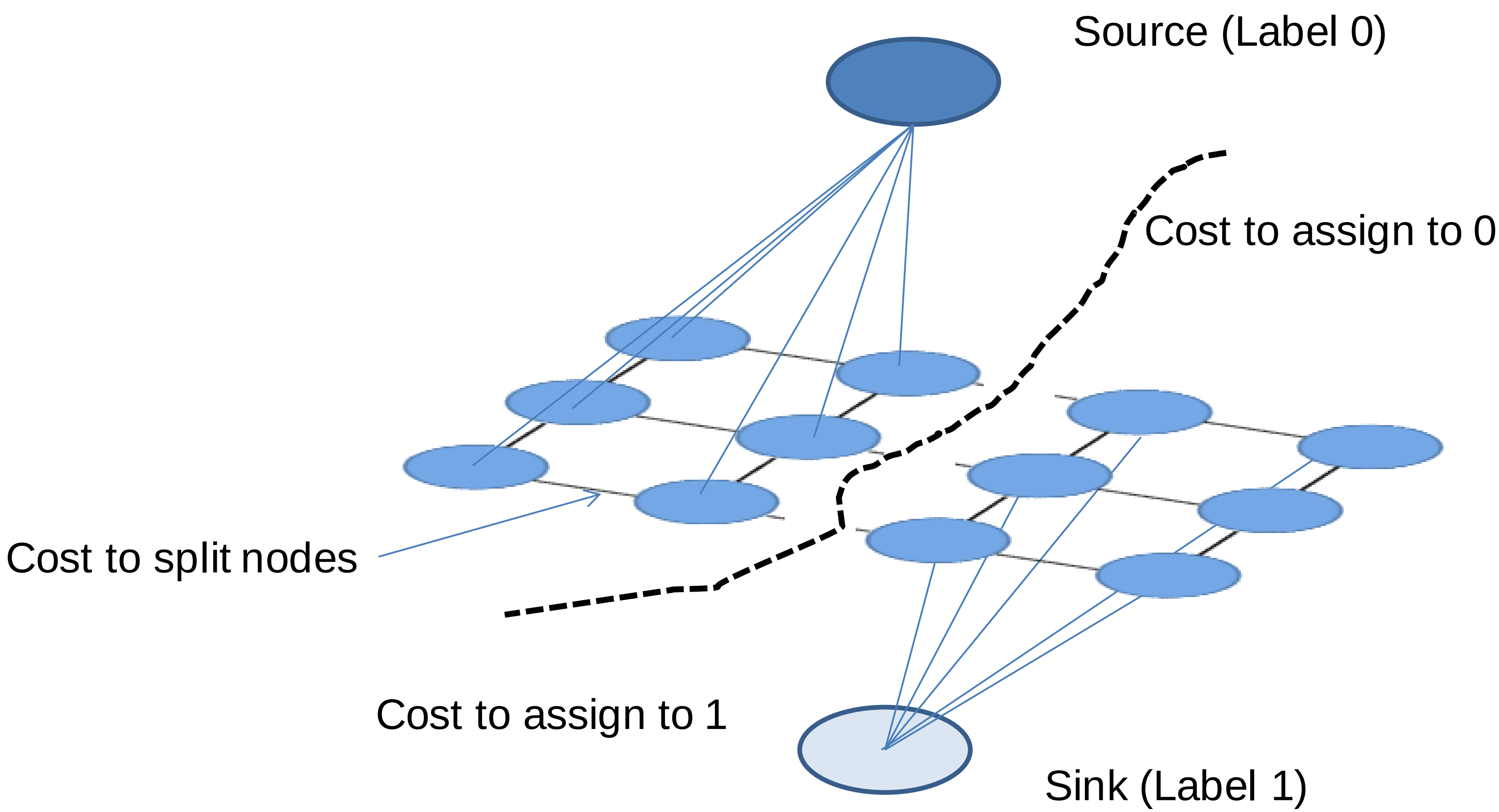
Solving MRFs with graph cuts



$$Energy(\mathbf{y};\theta,data)=\sum_i \psi_1(y_i;\theta,data)+\sum_{i,j\in edges}\psi_2(y_i,y_j;\theta,data)$$

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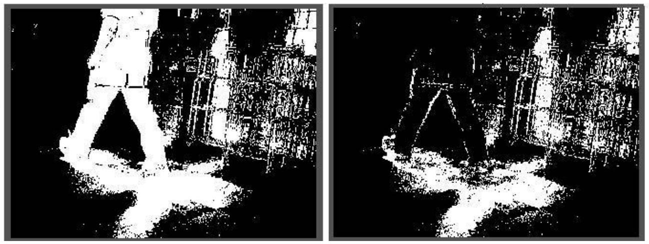
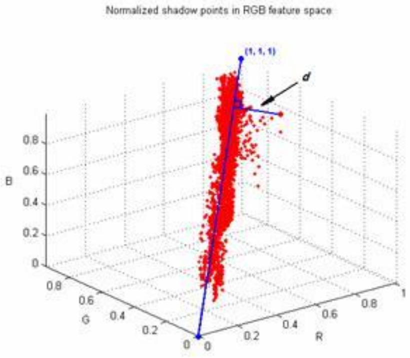
Foreground-Background segmentation

The code does the following:

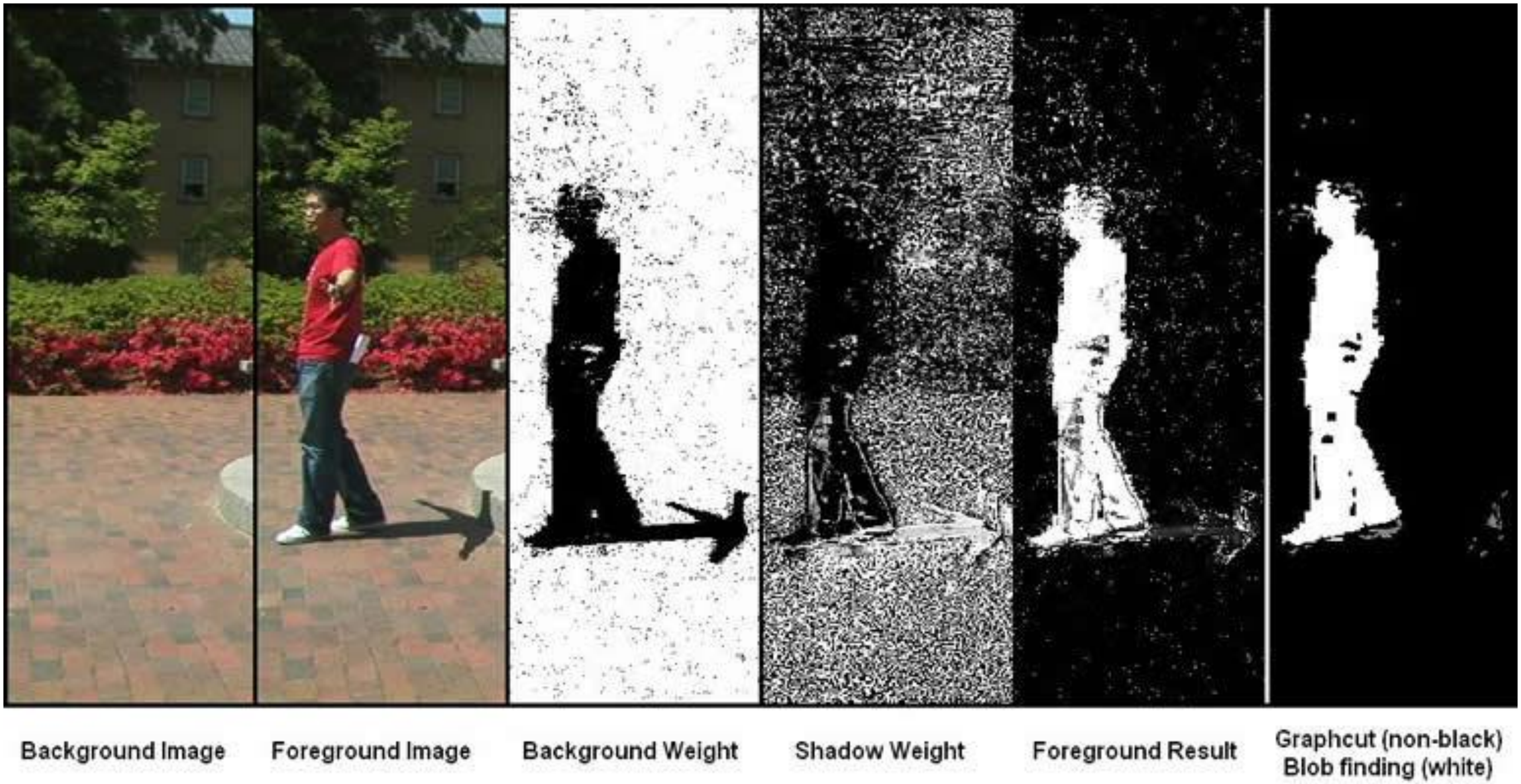
- background RGB Gaussian model training (from many images)
- shadow modeling (hard shadow & soft shadow).





- Graphcut foreground-background segmentation

Foreground-Background segmentation



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Foreground-Background segmentation

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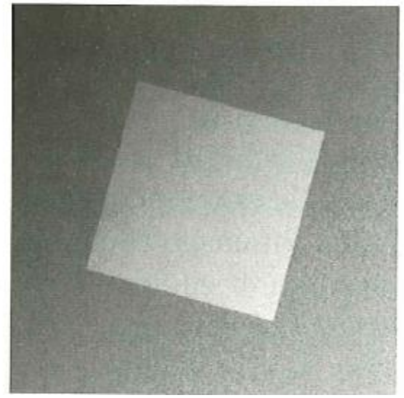
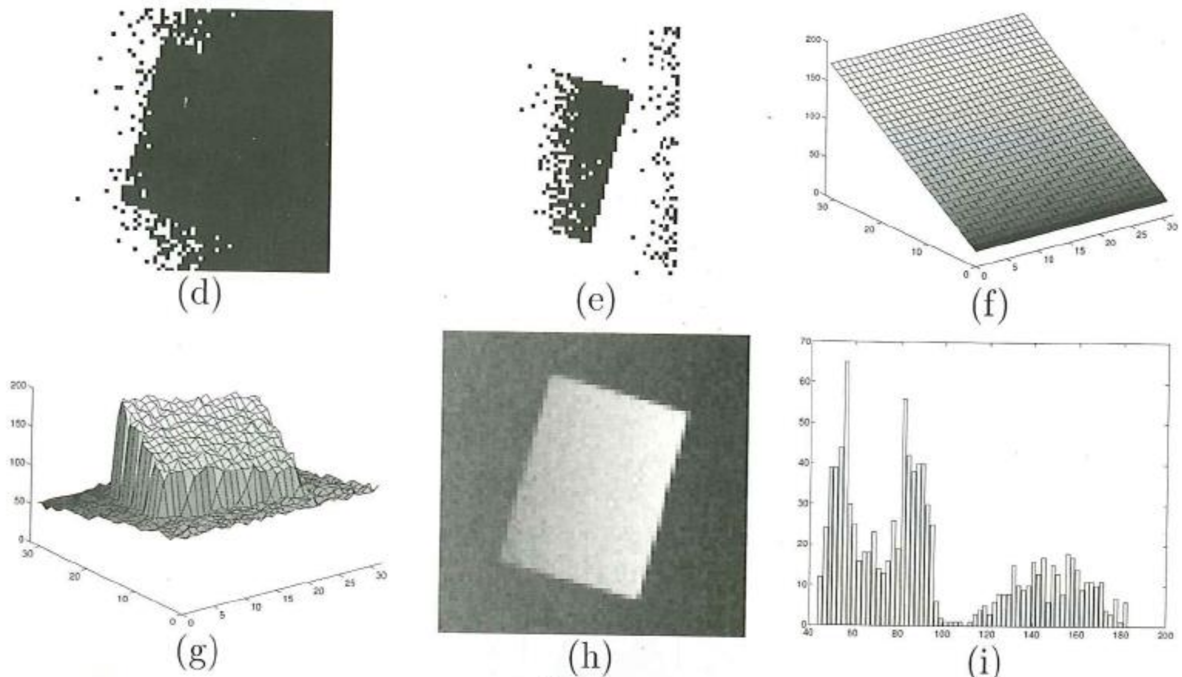


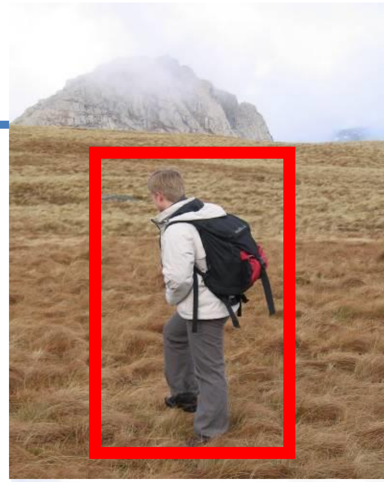
Inclusion criteria ?



e.g. $(a.I+b.x+c.y+d)^2$<threshold
(keep refitting a,b,c to included pts)

(j)

GrabCut – interactive foreground segmentation



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http://research.microsoft.com/en-us/people/carrot/

http://blogs.technet.com/b/office2010/archive/2009/11/30/more-about-background-removal-in-office-2010.aspx


MSR Cambridge Image Editing Technology - [JCPad1]
Ready

Problem

Fast &
Accurate ?

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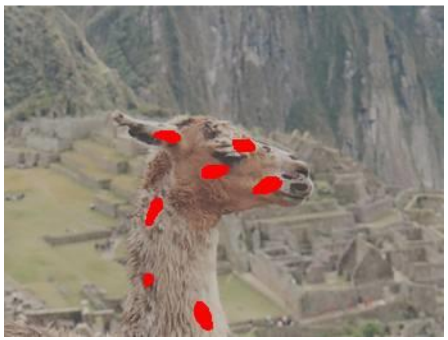
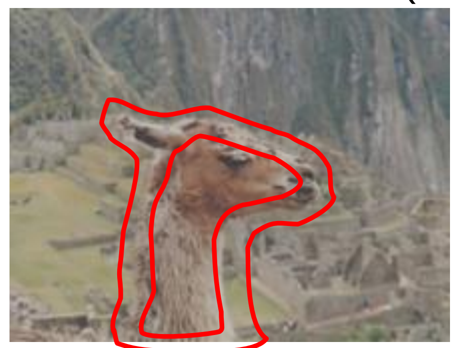
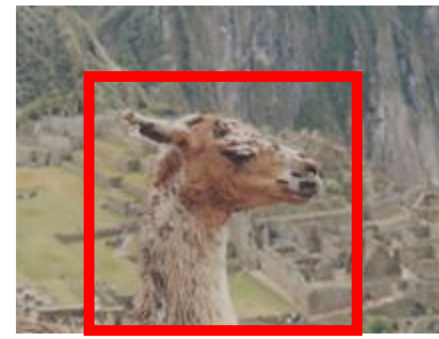
Slides from Carsten Rother (MSR)

What GrabCut does

	Magic Wand (198?)	Intelligent Scissors Mortensen and Barrett (1995)	GrabCut
User Input			
Result			
	Regions	Boundary	Regions & Boundary

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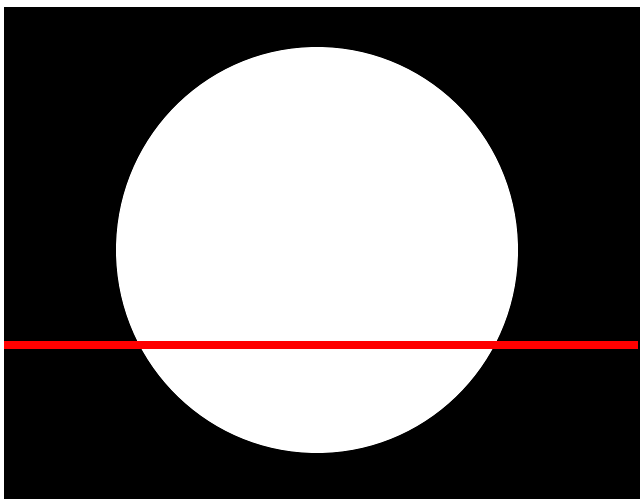
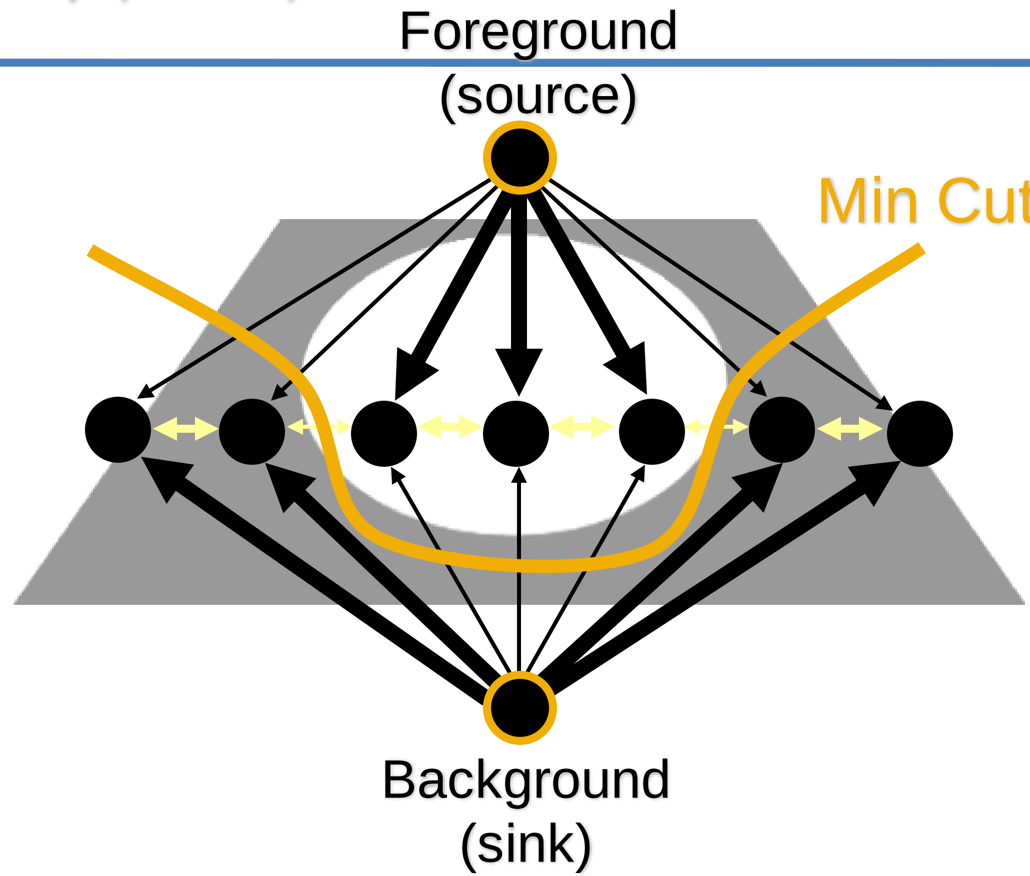
Graph Cuts

Boykov and Jolly (2001)

Image





Cut: separating source and sink; Energy: collection of edges

Min Cut: Global minimal energy in polynomial time

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Iterated Graph Cut

User Initialisation

K-means for learning colour distributions

Graph cuts to infer the segmentation

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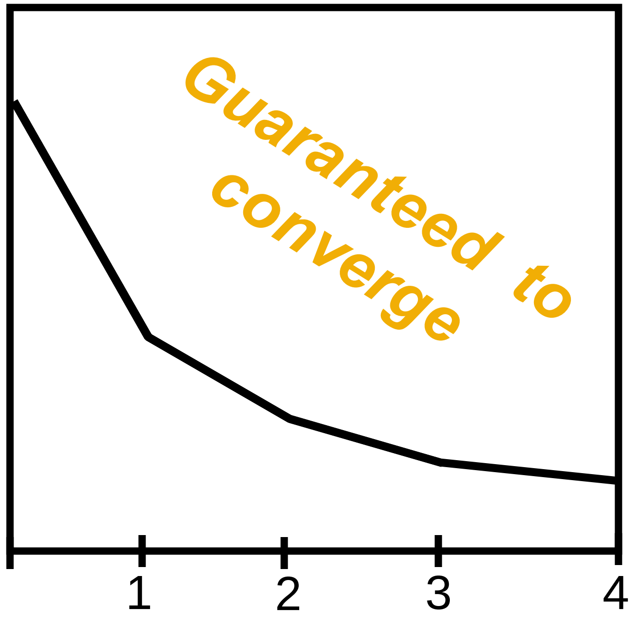
Iterated Graph Cuts



Result

Energy after each Iteration

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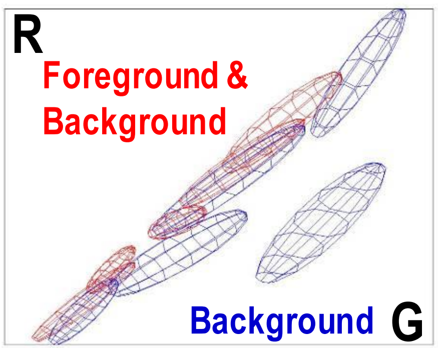
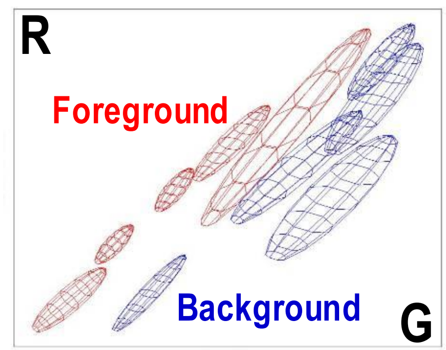
Colour Model



Iterated graph cut



Gaussian Mixture Model (typically 5-8 components)

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Moderately straightforward examples

… GrabCut completes automatically

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Difficult Examples

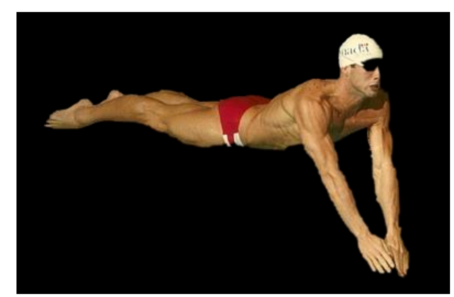
Camouflage & Low Contrast

Fine structure

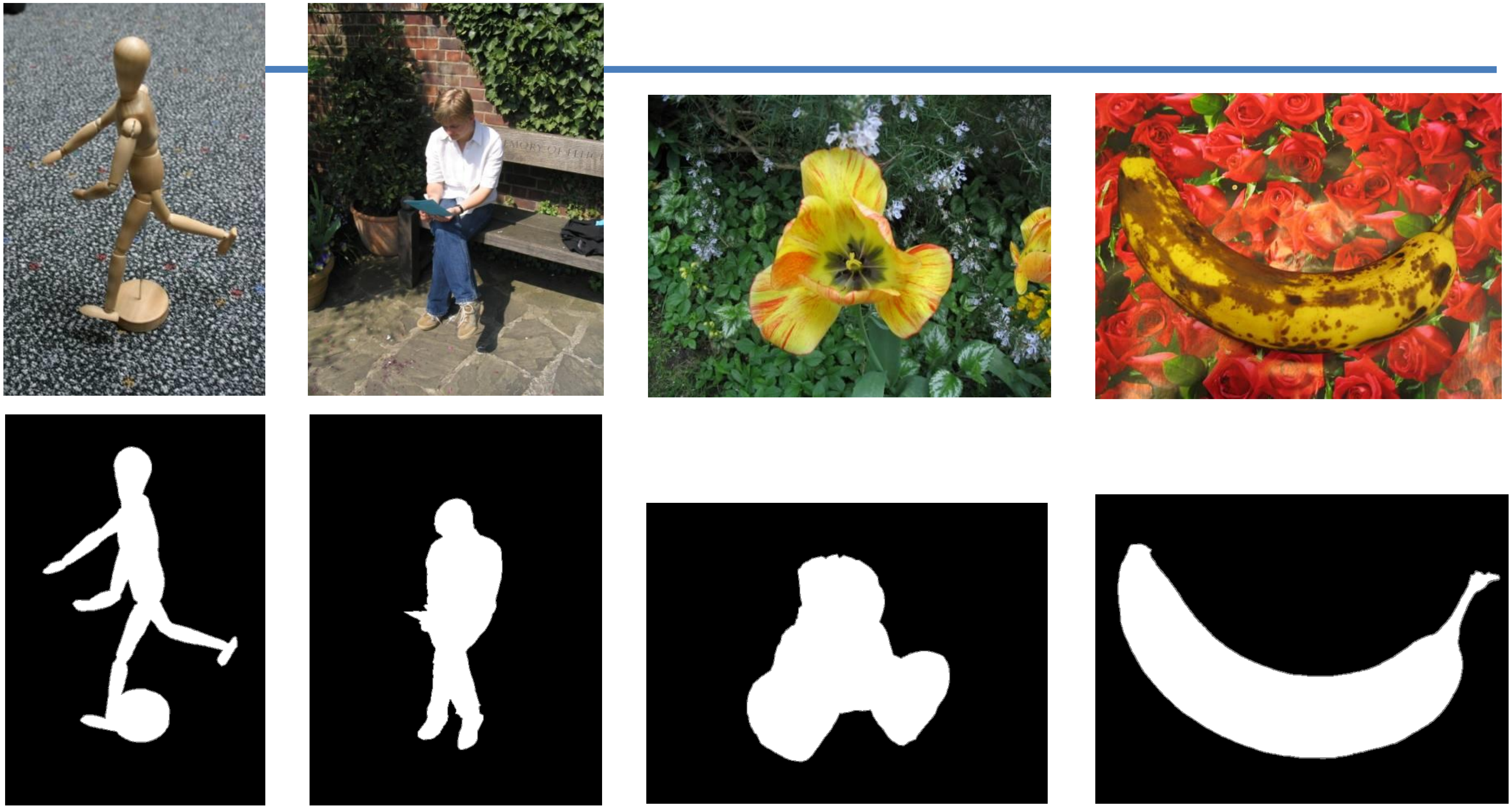
No telepathy

Initial Rectangle

Initial Result



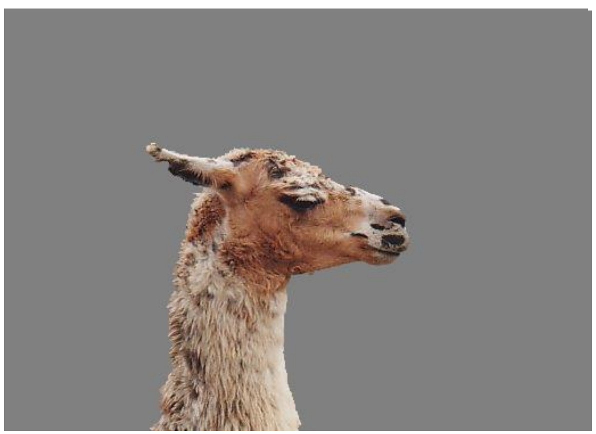
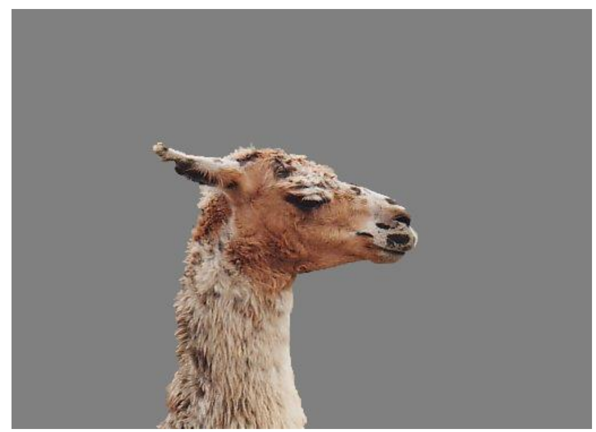
Evaluation – Labelled Database



Available online: http://research.microsoft.com/vision/cambridge/segmentation/

Slides from Carsten Rother (MSR)

Comparison

	Boykov and Jolly (2001)	GrabCut
User Input		
Result		
	Error Rate: 0.72%	**Error Rate: 0.72%**

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Border Matting

Hard Segmentation   Automatic Trimap   Soft Segmentation

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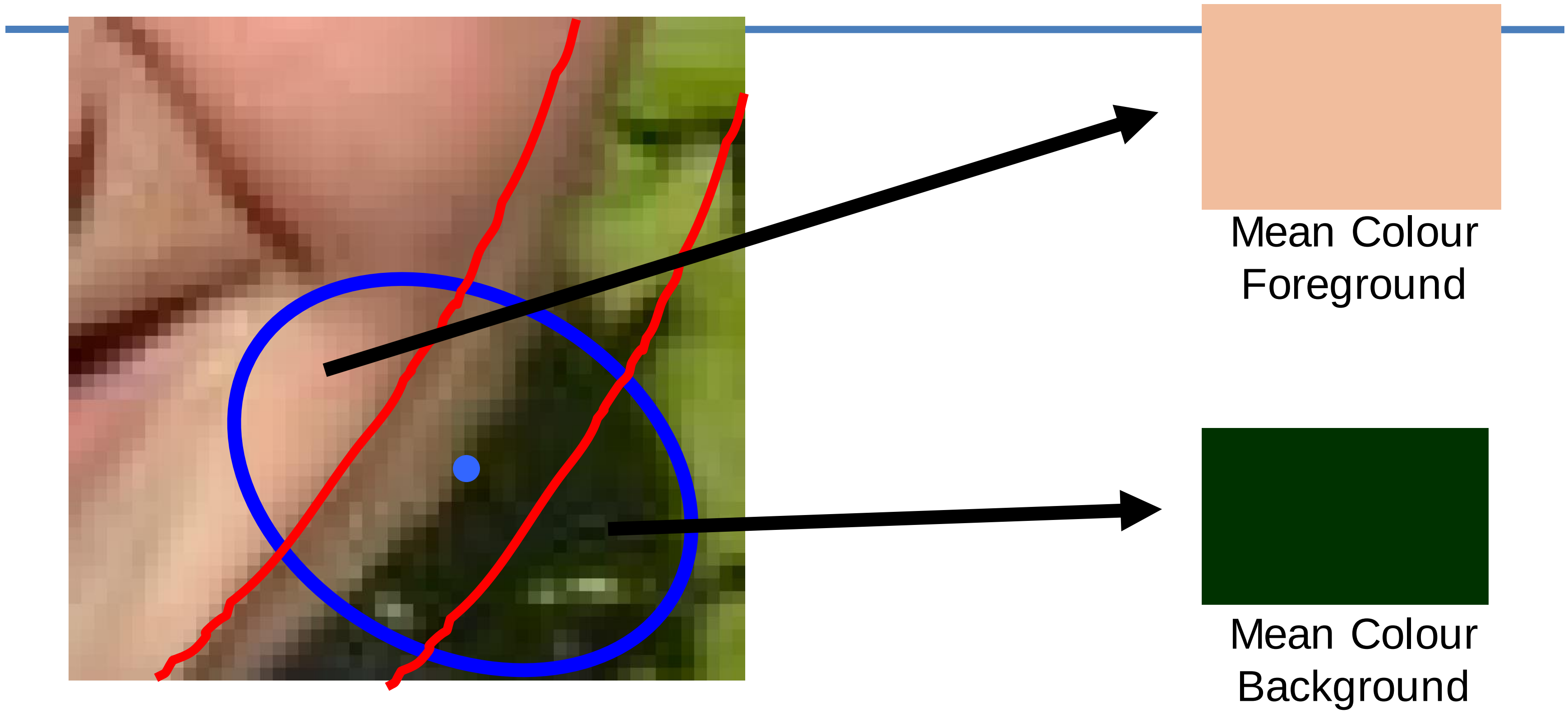
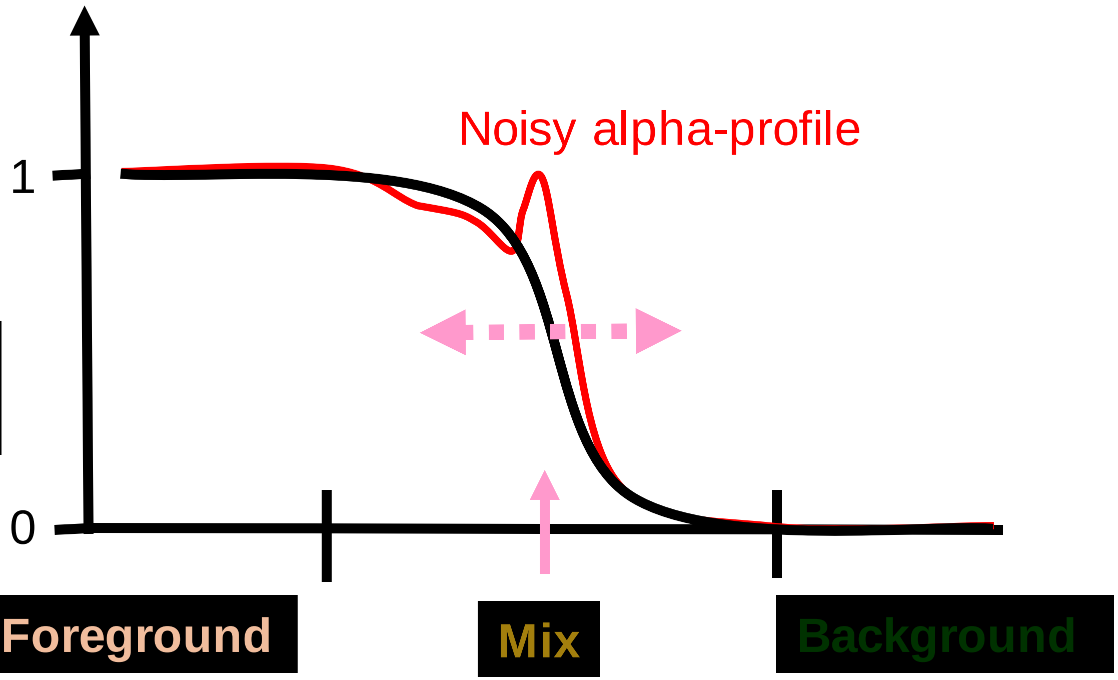
Natural Image Matting



Solve

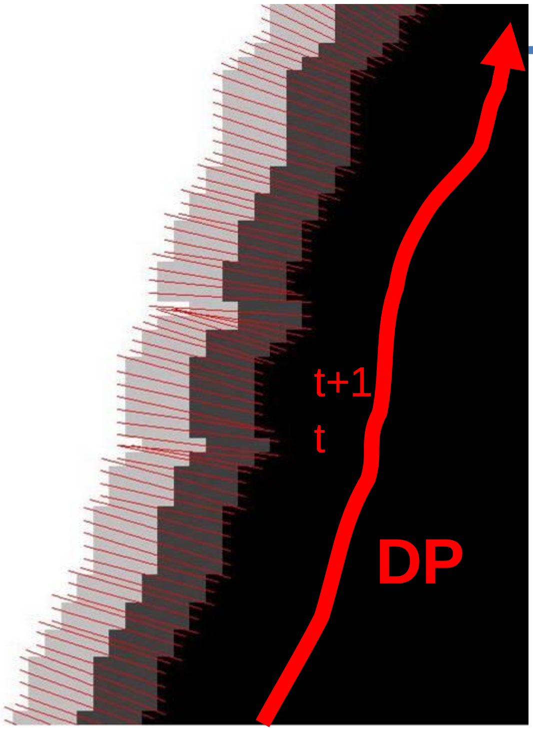
Ruzon and Tomasi (2000): Alpha estimation in natural images

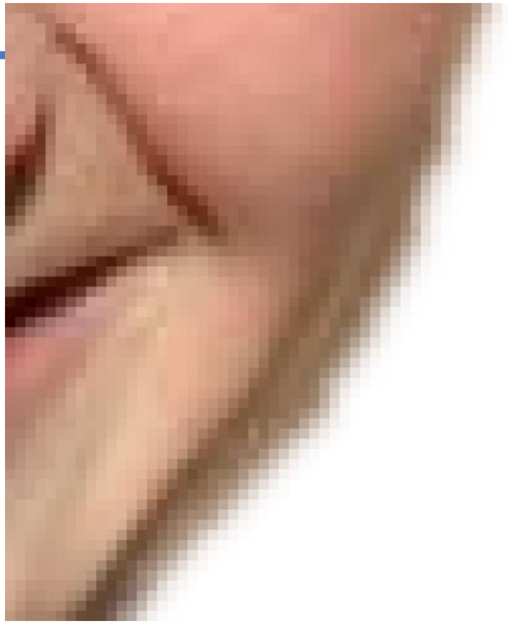
Border Matting



Fit a smooth alpha-profile with parameters

Dynamic Programming





Result using DP Border Matting

Noisy alpha-profile

Regularisation

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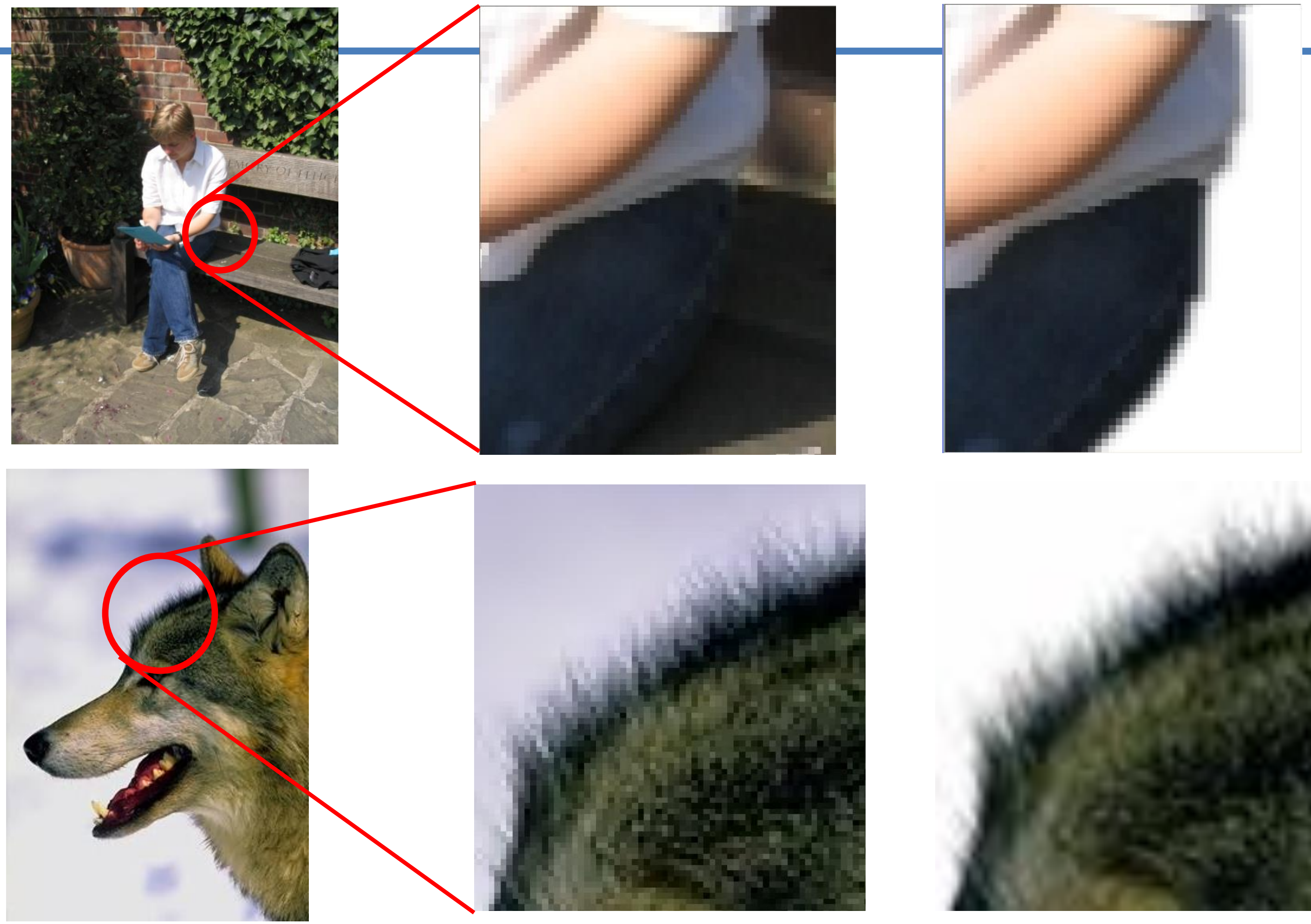
Results



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Switching to Spatial-domain only:

Morphological Operations

What Are Morphological Operators?

- Local pixel transformations for processing region shapes
- Most often used on binary images

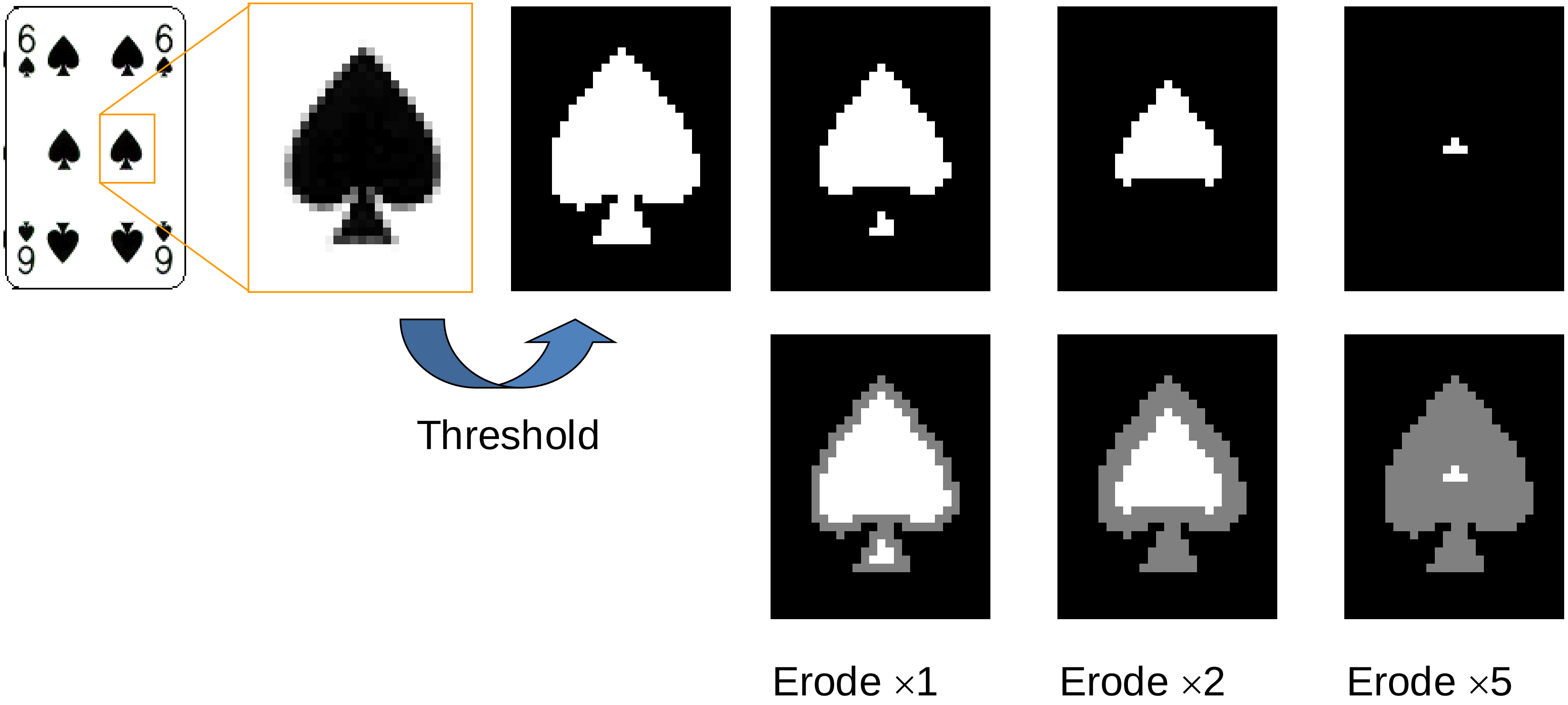
- Logical transformations based on comparison of pixel neighborhoods with a pattern.

Simple Operations - Examples

- Eight-neighbor erode
  - a.k.a. Minkowsky subtraction

- Erase any foreground pixel that has one eight-connected neighbor that is background.

8-neighbor erode

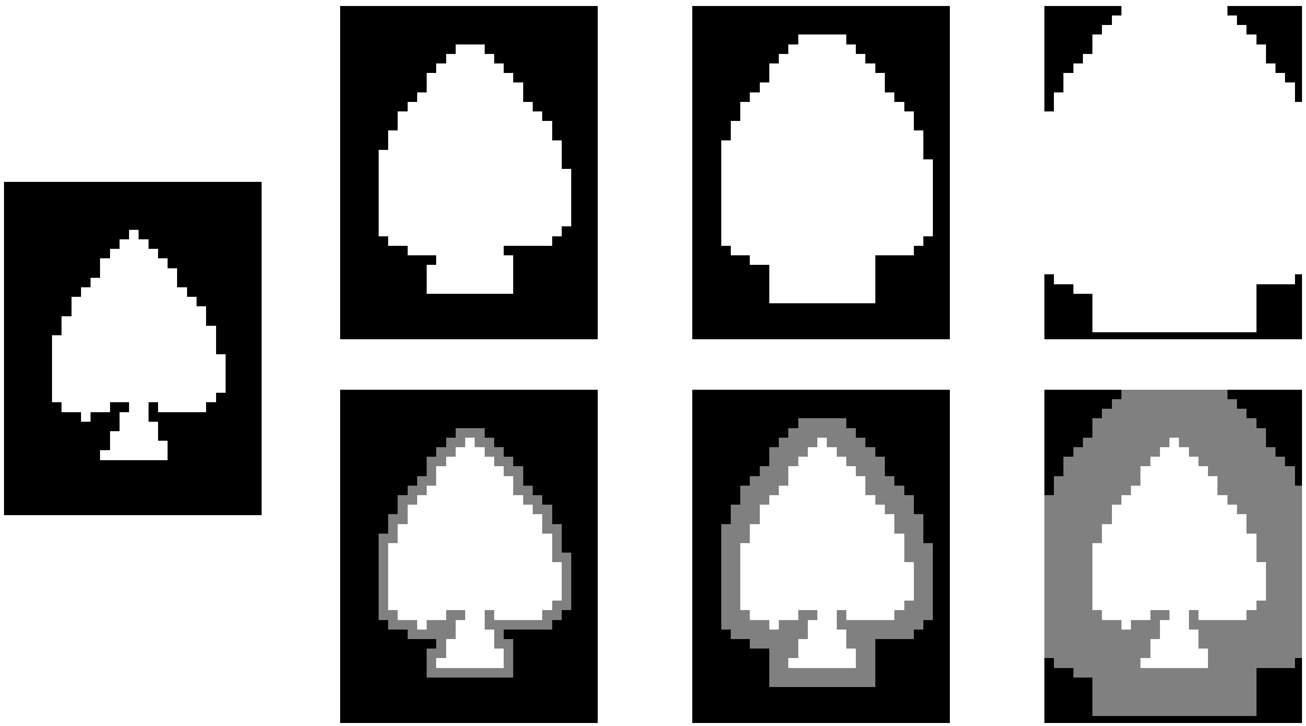


8-neighbor dilate

- Eight-neighbor dilate
  - a.k.a. Minkowsky addition

- Paint any background pixel that has one eight-connected neighbor that is foreground.

8-neighbor dilate



Dilate ×1 Dilate ×2 Dilate ×5

Why?

- Smooth region boundaries for shape analysis.
- Remove noise and artefacts from an imperfect segmentation.

Structuring Elements

- Morphological operations take two arguments:
    - A binary image
    - A *structuring element.*
- Compare the structuring element to the neighborhood of each pixel.
- This determines the output of the morphological operation.

Structuring elements

- The structuring element is also a binary array
- A structuring element has an origin

1	1	1
1	1	1
1	1	1

0	1	0
1	1	1
0	1	0

0	1	0	1
1	0	1	1
0	1	0	0

Binary images as sets

- We can think of the binary image and the structuring element as sets containing the pixels with value 1.

1	1	1	0	0
0	1	1	0	0
0	0	0	0	0
0	0	0	1	0
0	0	0	0	0

$I = \{(1,1), (2,1), (3,1), (2,2), (3,2), (4,4)\}$

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Some set notation

- Union and intersection:

$$I_1 \cup I_2 = \{\underline{x} : \underline{x} \in I_1 \text{ or } \underline{x} \in I_2\}$$
$$I_1 \cap I_2 = \{\underline{x} : \underline{x} \in I_1 \text{ and } \underline{x} \in I_2\}$$

- Complement

$$I^C = \{\underline{x} : \underline{x} \notin I\}$$

- Difference

$$I_1 \setminus I_2 = \{\underline{x} : \underline{x} \in I_1 \text{ and } \underline{x} \notin I_2\}$$

- We use ϕ for the empty set .

Fitting, Hitting and Missing

- *S* fits *I* at $\underline{x}$ if

  $$\{\underline{y} : \underline{y} = \underline{x} + \underline{s}, \underline{s} \in S\} \subset I$$

- *S* hits *I* at $\underline{x}$ if

  $$\{\underline{y} : \underline{y} = \underline{x} - \underline{s}, \underline{s} \in S\} \cap I \neq \phi$$

- *S* misses *I* at $\underline{x}$ if

  $$\{\underline{y} : \underline{y} = \underline{x} - \underline{s}, \underline{s} \in S\} \cap I = \phi$$

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Fitting, Hitting and Missing

Image

0	1	0	1	1	1	1	1
1	0	1	1	1	1	1	1
0	1	1	1	0	0	0	0
0	1	1	1	0	0	0	0
0	1	1	1	1	0	0	0
0	1	0	1	0	1	0	0
0	1	0	0	0	0	1	0
0	1	0	0	0	0	0	0

Structuring element

0	1	0
1	1	1
0	1	0

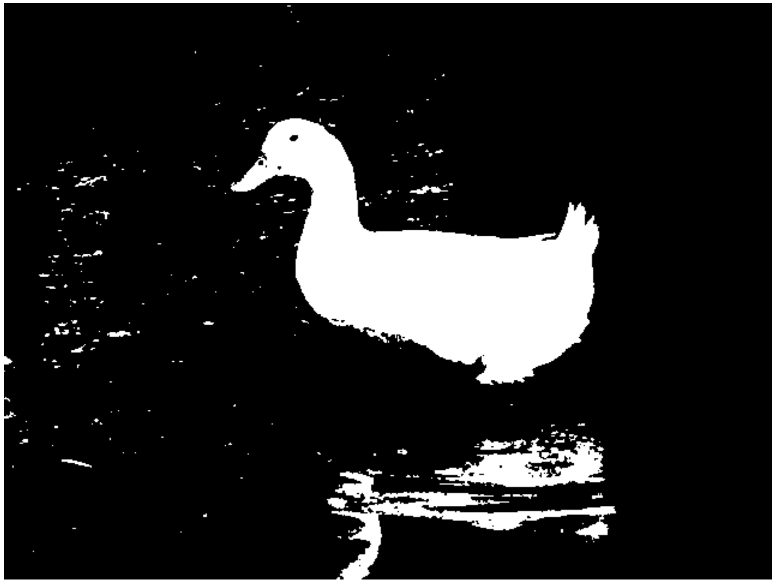
Erosion

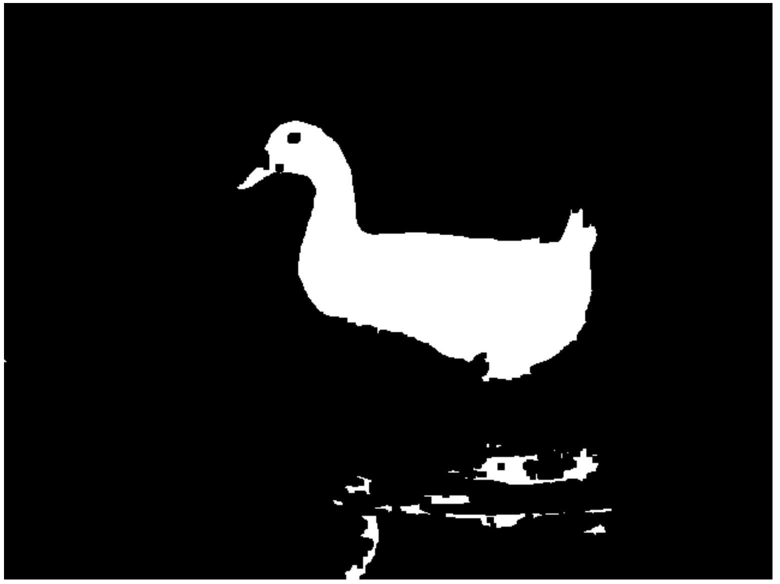
- The image $E = I \ominus S$ is the *erosion* of image *I* by structuring element *S*.

$$E(\underline{x}) = \begin{cases} 1 & \text{if } S \text{ fits } I \text{ at } \underline{x} \\ 0 & \text{otherwise} \end{cases}$$

$$E = \{\underline{x} : \underline{x} + \underline{s} \in I \text{ for every } s \in S\}$$

Example

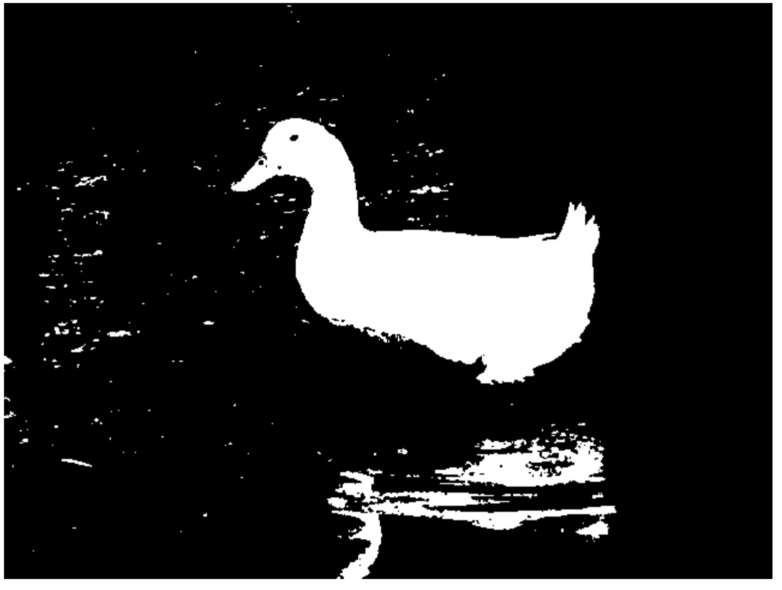


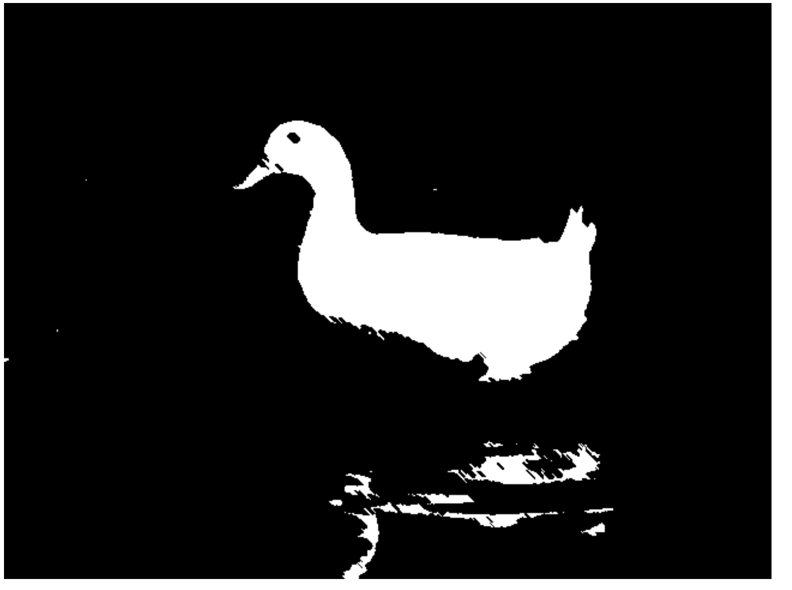


Structuring element

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Example





Structuring element

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

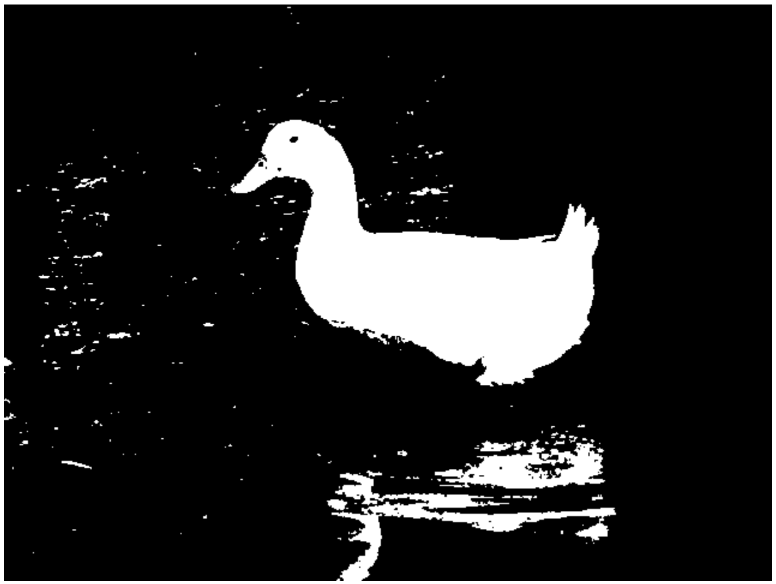
Dilation

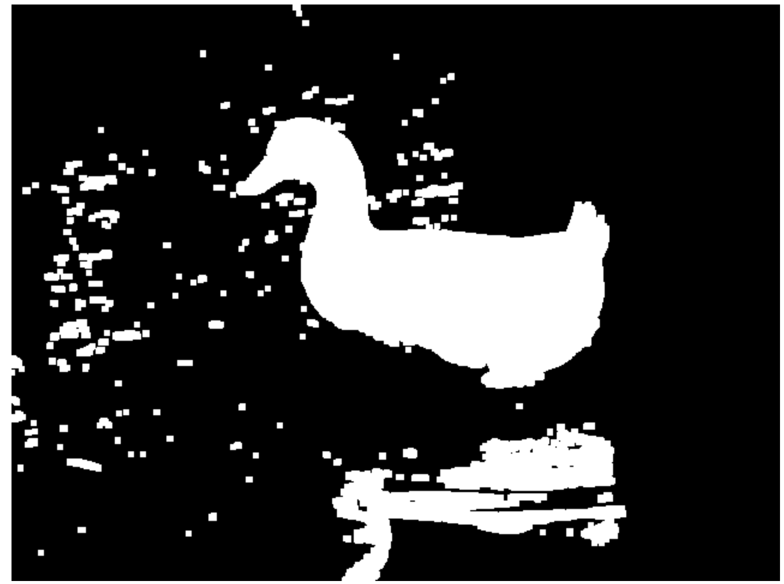
- The image $D = I \oplus S$ is the *dilation* of image I by structuring element S.

$$D(\underline{x}) = \begin{cases} 1 & \text{if } S \text{ hits } I \text{ at } \underline{x} \\ 0 & \text{otherwise} \end{cases}$$

$$D = \left\{ \underline{x} : \underline{x} - \underline{s},\ \underline{y} \in I \text{ and } \underline{s} \in S \right\}$$

Example

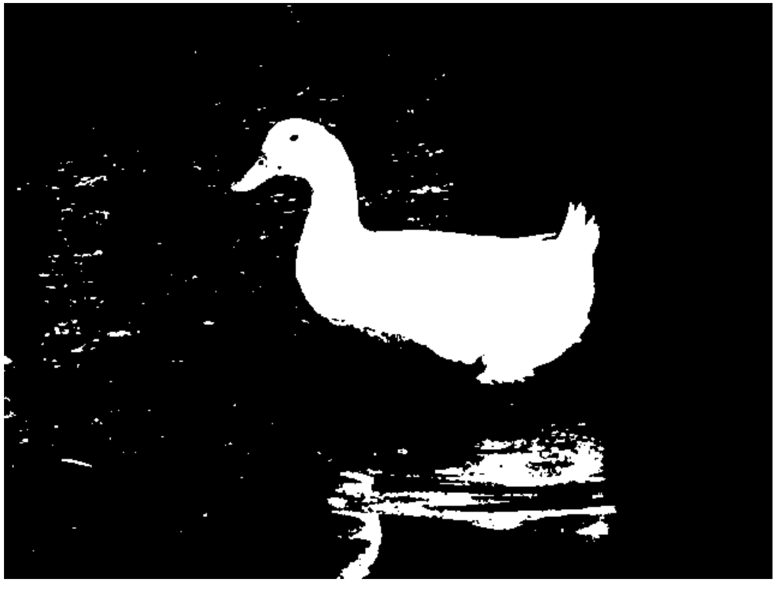


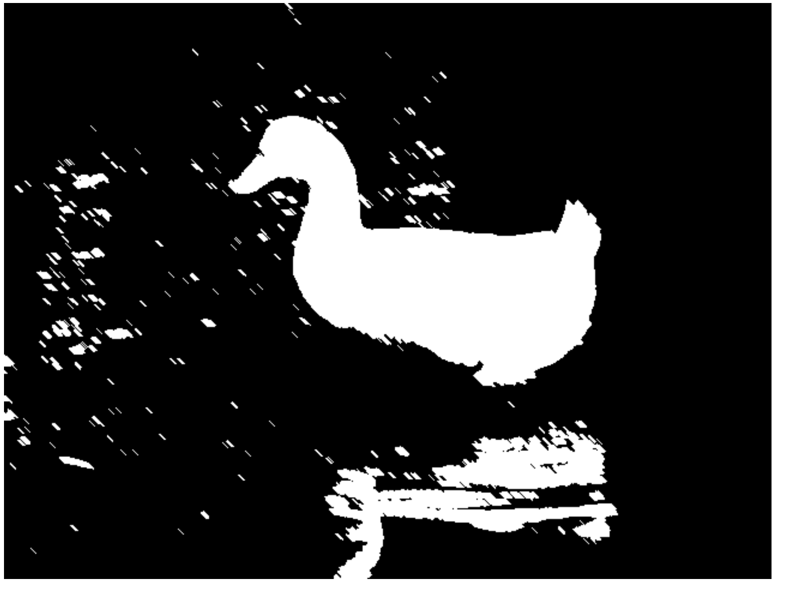


Structuring element

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Example





Structuring element

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Opening and Closing

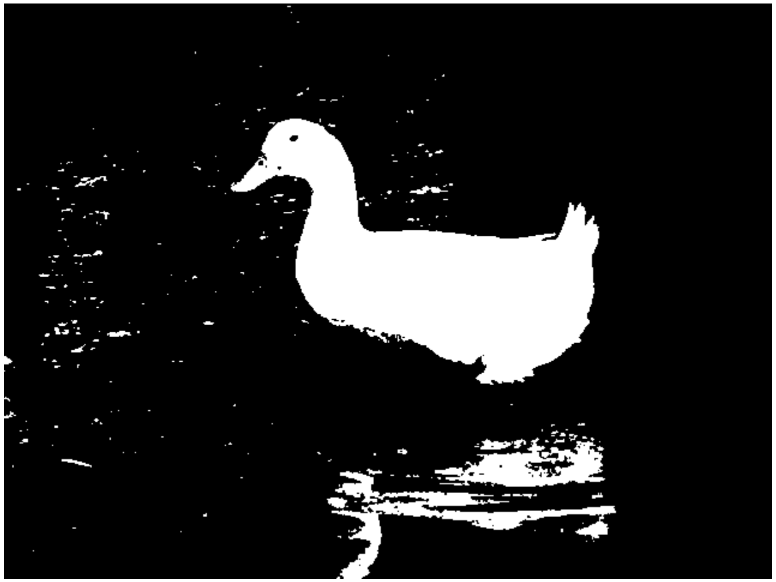
- The *opening* of *I* by *S* is

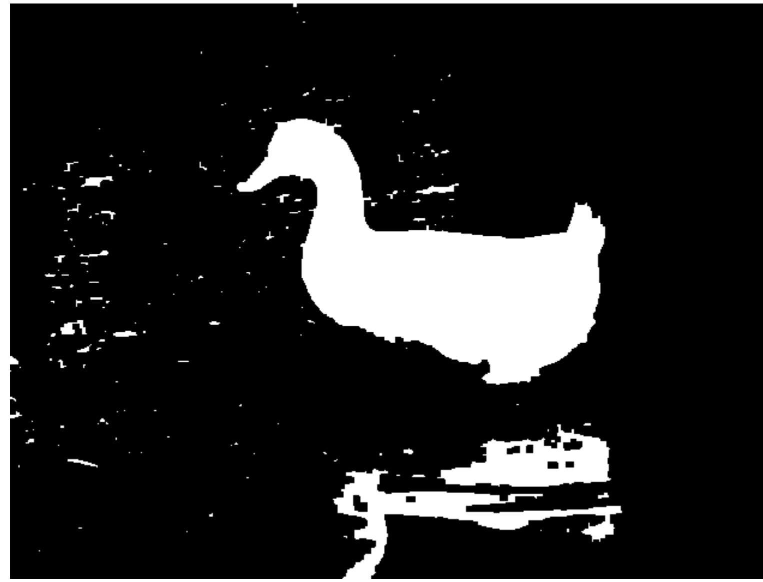
$$I \circ S = (I \ominus S) \oplus S$$

- The *closing* of *I* by *S* is

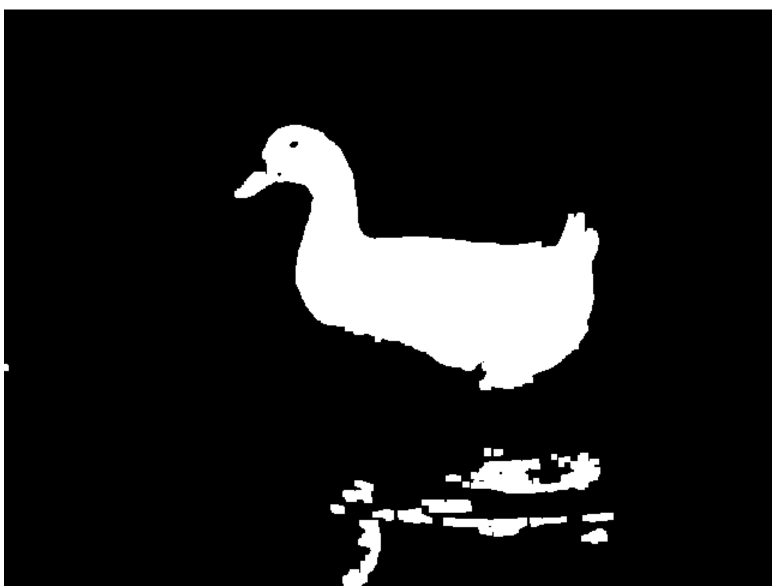
$$I \bullet S = (I \oplus S) \ominus S$$

Example





close



open

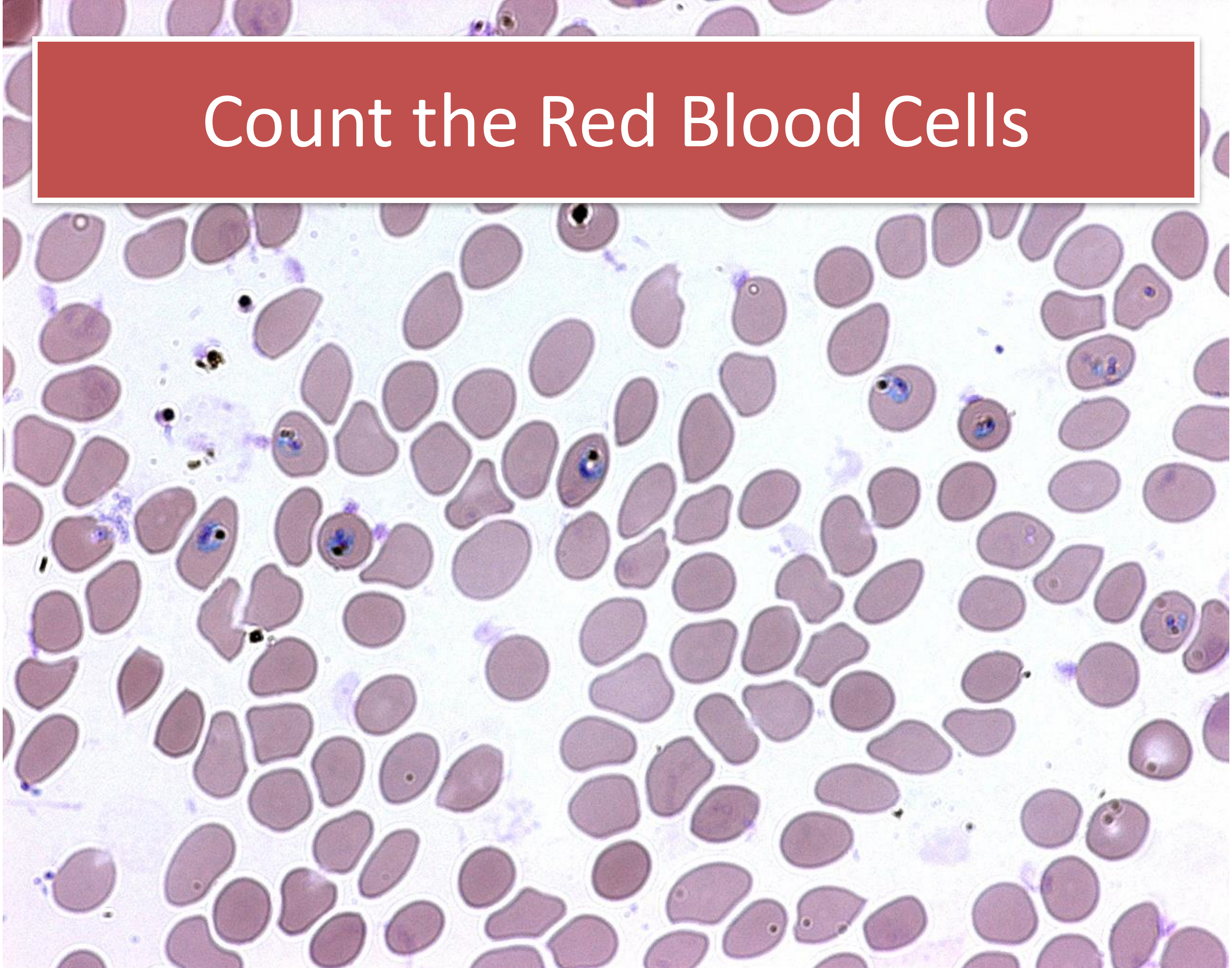
Structuring element

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Morphological filtering

- To remove holes in the foreground and islands in the background, do both opening and closing.
- The size and shape of the structuring element determine which features survive.
- In the absence of knowledge about the shape of features to remove, use a circular structuring element.

Count the Red Blood Cells



Granulometry

- Provides a size distribution of distinct regions or “granules” in the image.
- We open the image with increasing structuring element size and count the number of regions after each operation.

- Creates a “morphological sieve”

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Granulometry

```
def granulo(I, T, maxRad):
B = (I > T) # Segment the image I
# Open the image at each structuring element size up to a
# maximum and count the remaining regions.
numRegions = []
for x in range(1, maxRad + 1):
  kernel = cv2.getStructuringElement(cv2.MORPH_ELLIPSE,(x, x))
  O = cv2.morphologyEx(B, cv2.MORPH_OPEN, kernel)
  numComponents, _ = cv2.connectedComponents(O)
  numRegions.append(numComponents)
return numRegions
```

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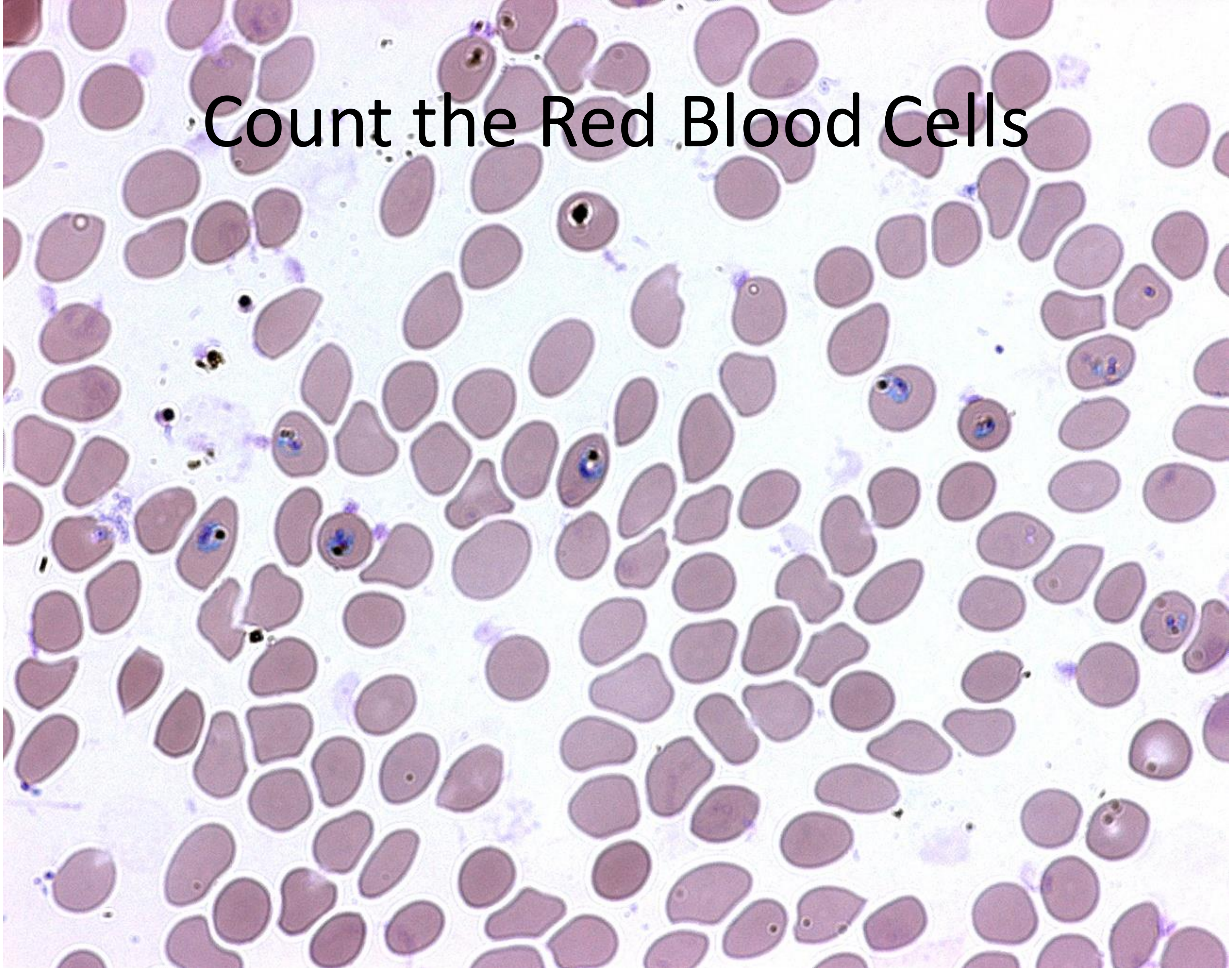
Count the Red Blood Cells



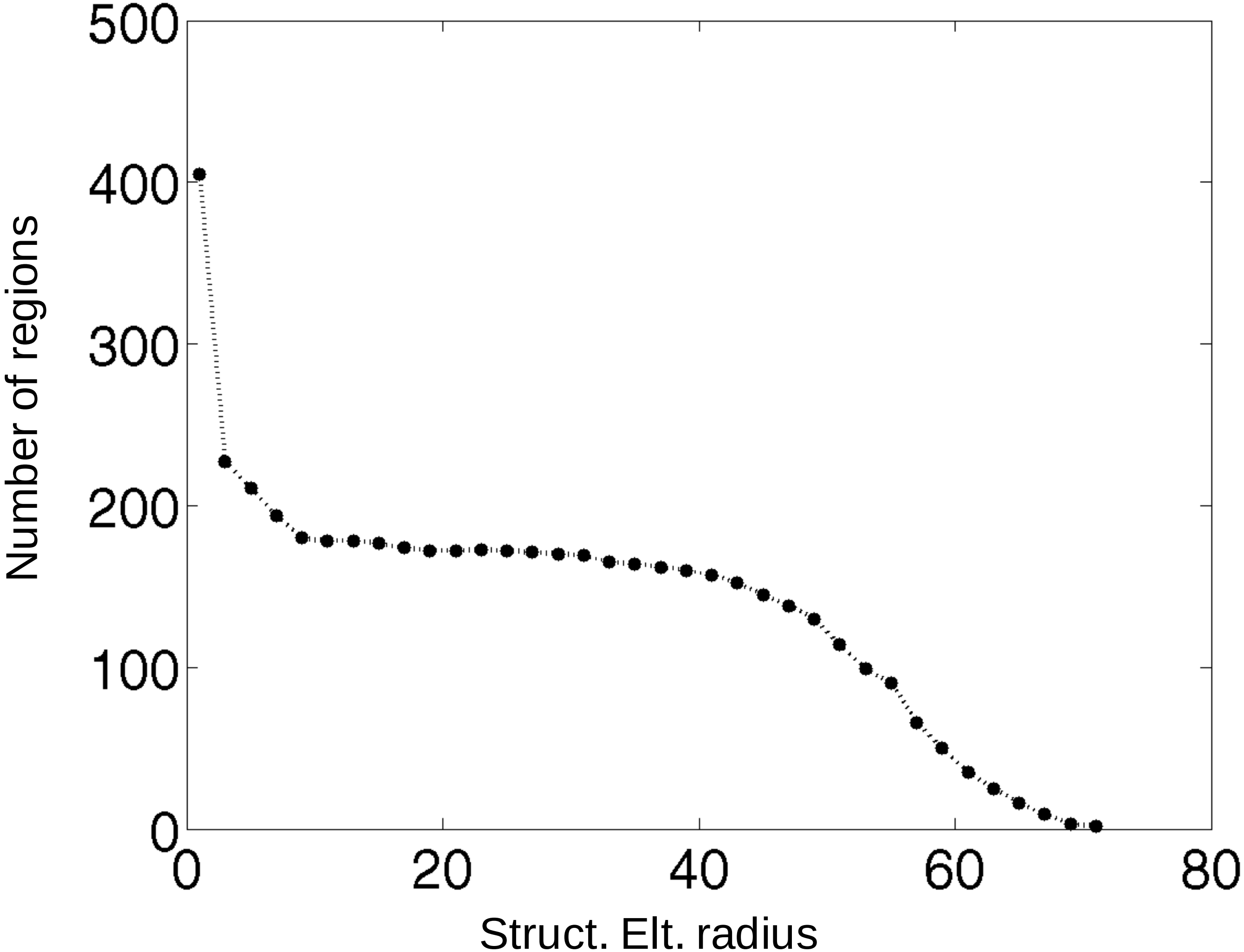
Threshold and Label


Disc(11)

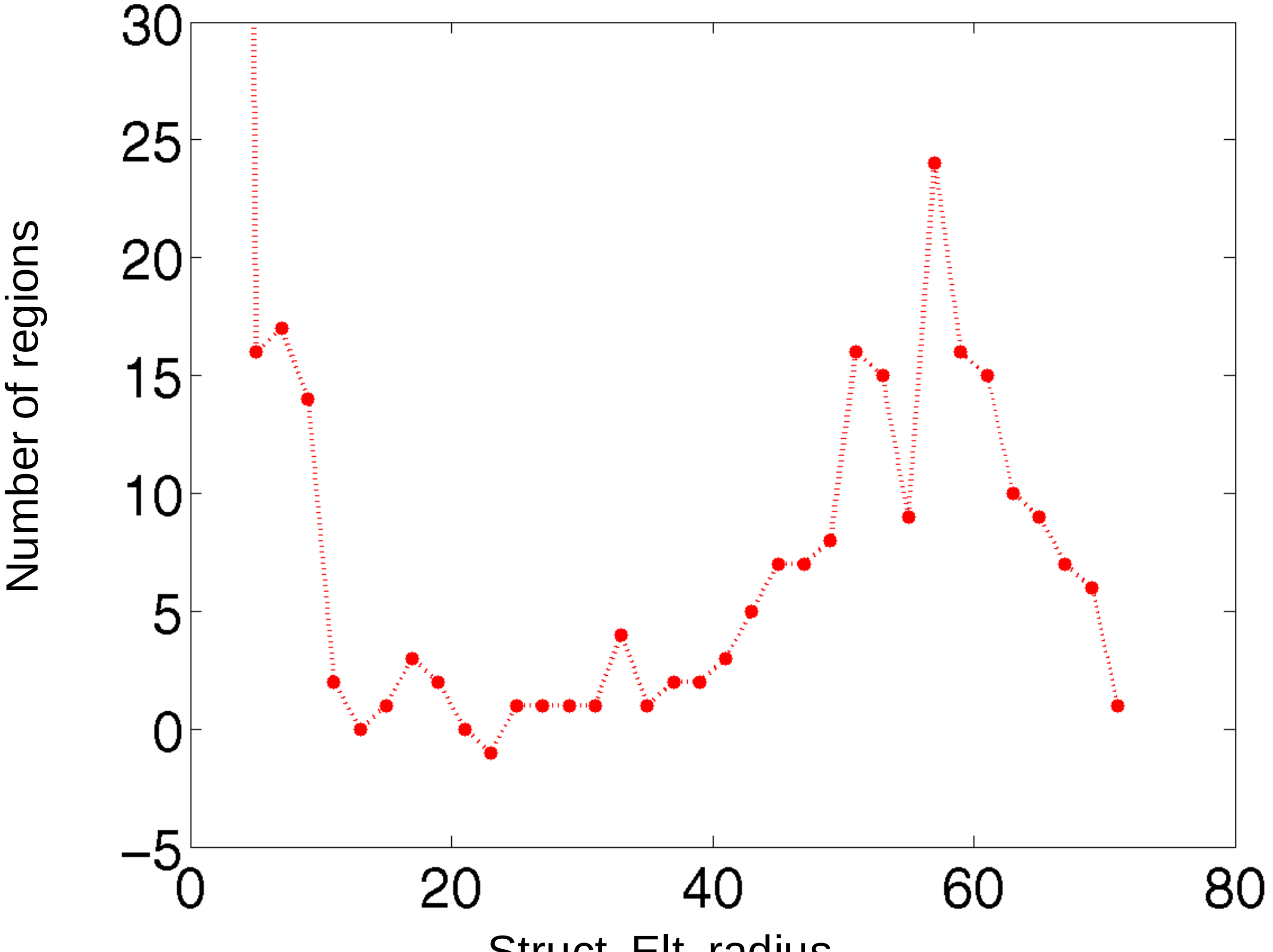

Disc(19)


Disc(59)

Number of Regions



Granulometric *Pattern Spectrum*



Hit-and-miss transform

- Searches for an exact match of the structuring element.
- $H = I \otimes S$ is the hit-and-miss transform of image *I* by structuring element *S*.
- Simple form of template matching.

Hit-and-miss transform

1	0	1	1	1
1	1	0	1	0
1	0	0	1	1
1	1	0	1	1
1	0	1	0	1

$\otimes$

1	0	1

=

0	1	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0

1	0	1	1	1
1	1	0	1	0
1	0	0	1	1
1	1	0	1	1
1	0	1	0	1

$\otimes$

1	0
*	1

=

0	1	0	0	0
0	0	0	0	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

Upper-Right Corner Detector

x	0	0
1	1	0
x	1	x

0	0	0
1	1	0
0	1	0

J

0	1	1
0	0	1
0	0	0

K

Thinning and Thickening

- Defined in terms of the hit-and-miss transform:
- The *thinning* of *I* by *S* is

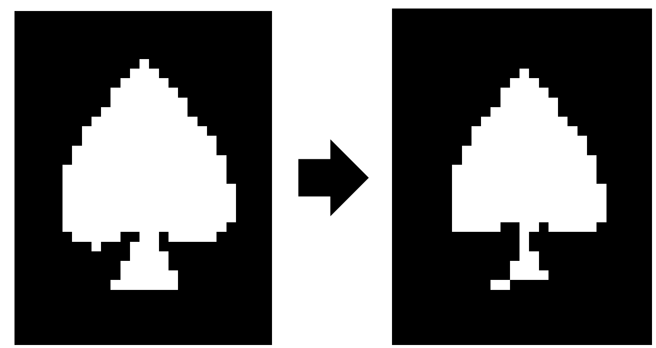
$$I \oslash S = I \setminus (I \otimes S)$$

- The *thickening* of *I* by *S* is

$$I \odot S = I \cup (I \otimes S)$$

- Dual operations:

$$(I \odot S)^C = I^C \oslash S$$



0	0	0
	1	
1	1	1

	0	0
1	1	0
	1	

Sequential Thinning/Thickening

- These operations are often performed in sequence with a selection of structuring elements S_1, S_2, ..., S_n.
- Sequential thinning:

$$I \oslash \{S_i : i = 1, \ldots, n\} = (((I \oslash S_1) \oslash S_2) \ldots \oslash S_n)$$

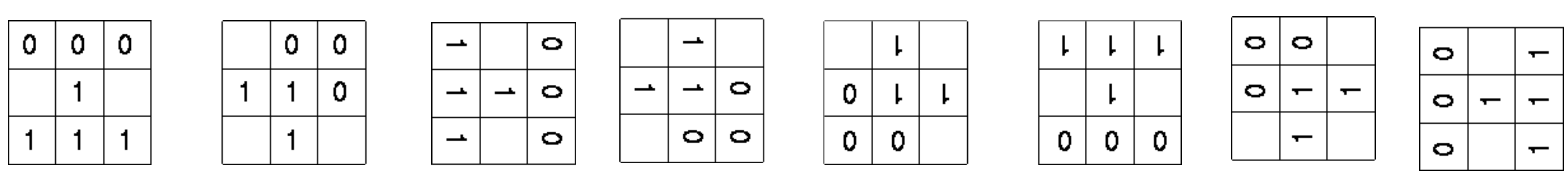
- Sequential thickening:

$$I \odot \{S_i : i = 1, \ldots, n\} = (((I \odot S_1) \odot S_2) \ldots \odot S_n)$$

Sequential Thinning/Thickening

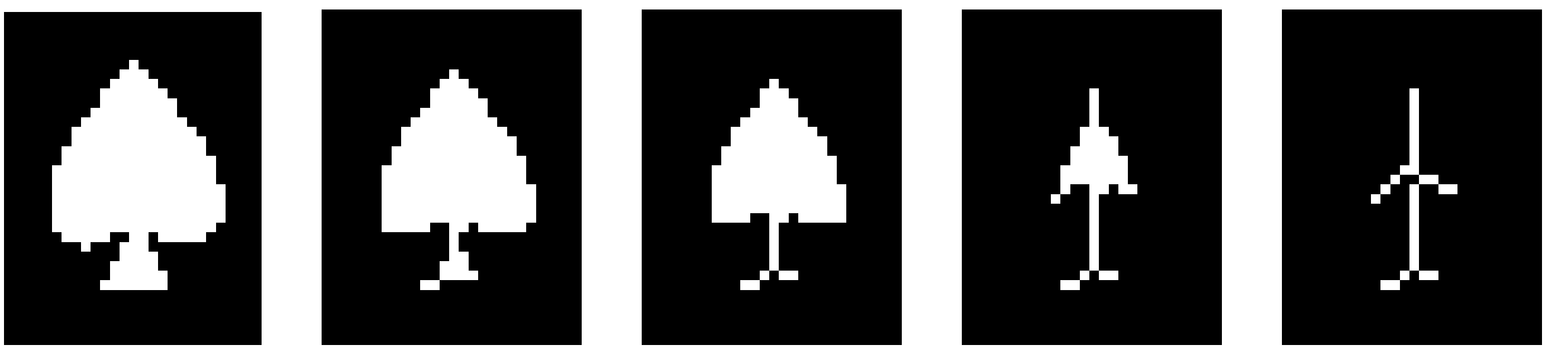
- Several sequences of structuring elements are useful in practice
- These are usually the set of rotations of a single structuring element.

- Sometimes called the *Golay alphabet.*



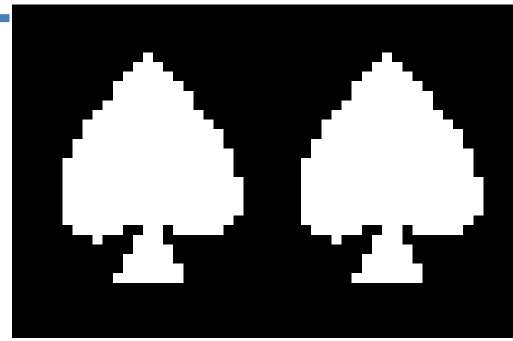
Sequential Thinning

- See *morphologyEx* in python.

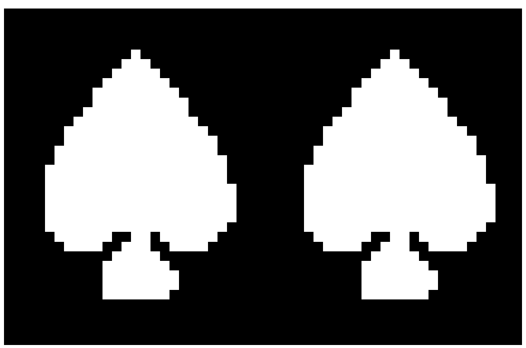


0 iterations | 1 iteration | 2 iterations | 5 iterations | Inf iterations

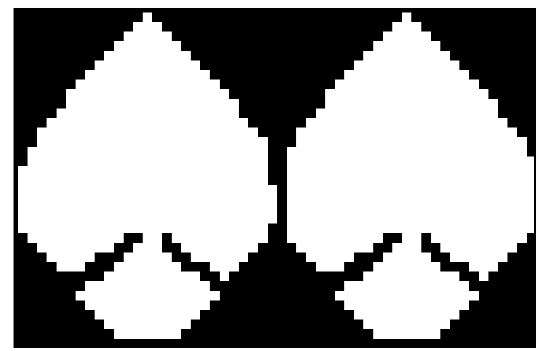
Sequential Thickening



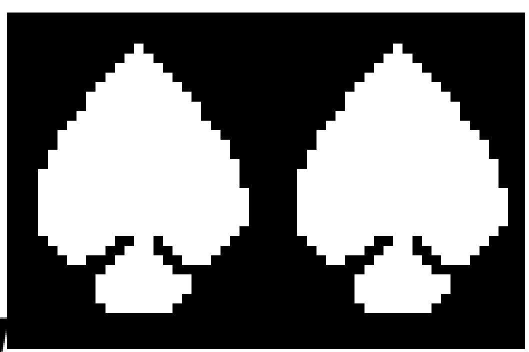
1
iteration



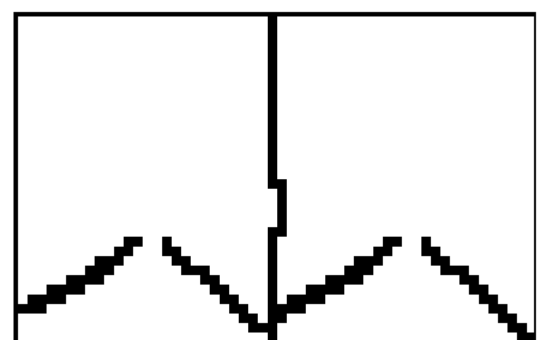
5
iterations



2
iterations



Inf
iterations

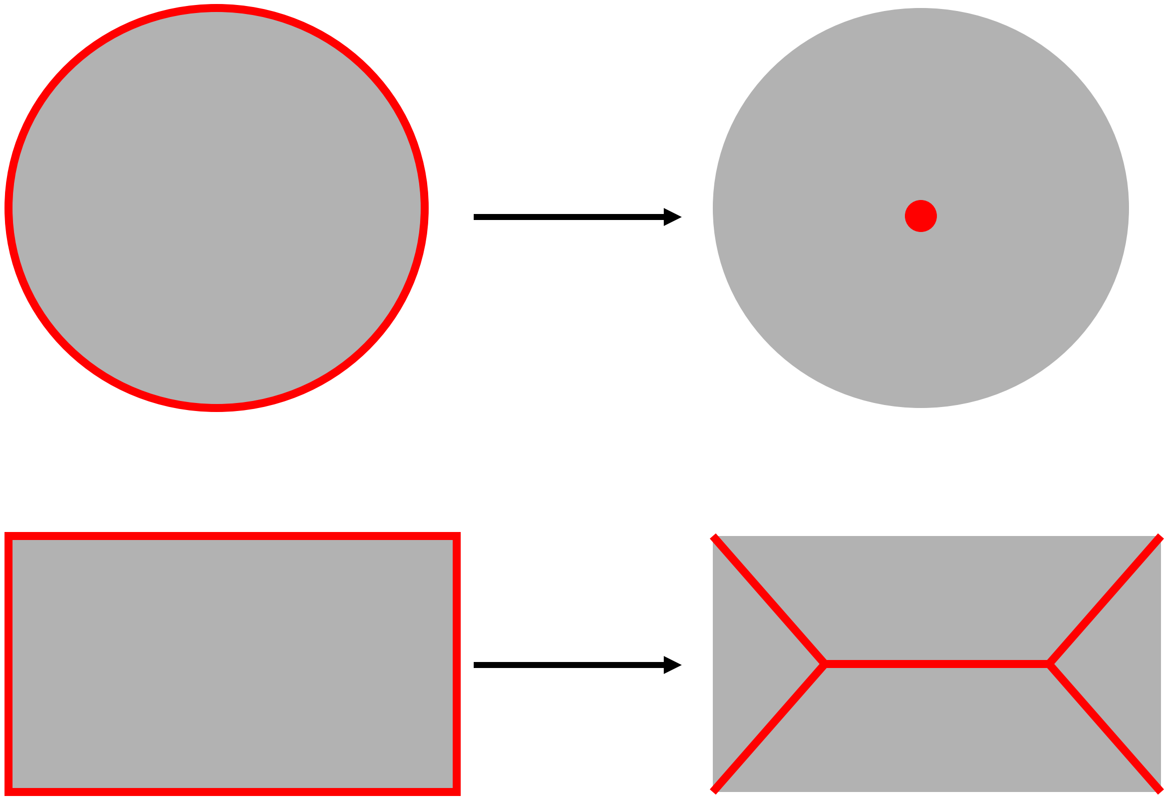


Skeletonization and the Medial Axis Transform

- The skeleton and *medial axis transform* (MAT) are stick-figure representations of a region $X \subset \Re^2$.

- Start a grassfire at the boundary of the region.
- The skeleton is the set of points at which two fire fronts meet.

Skeletons

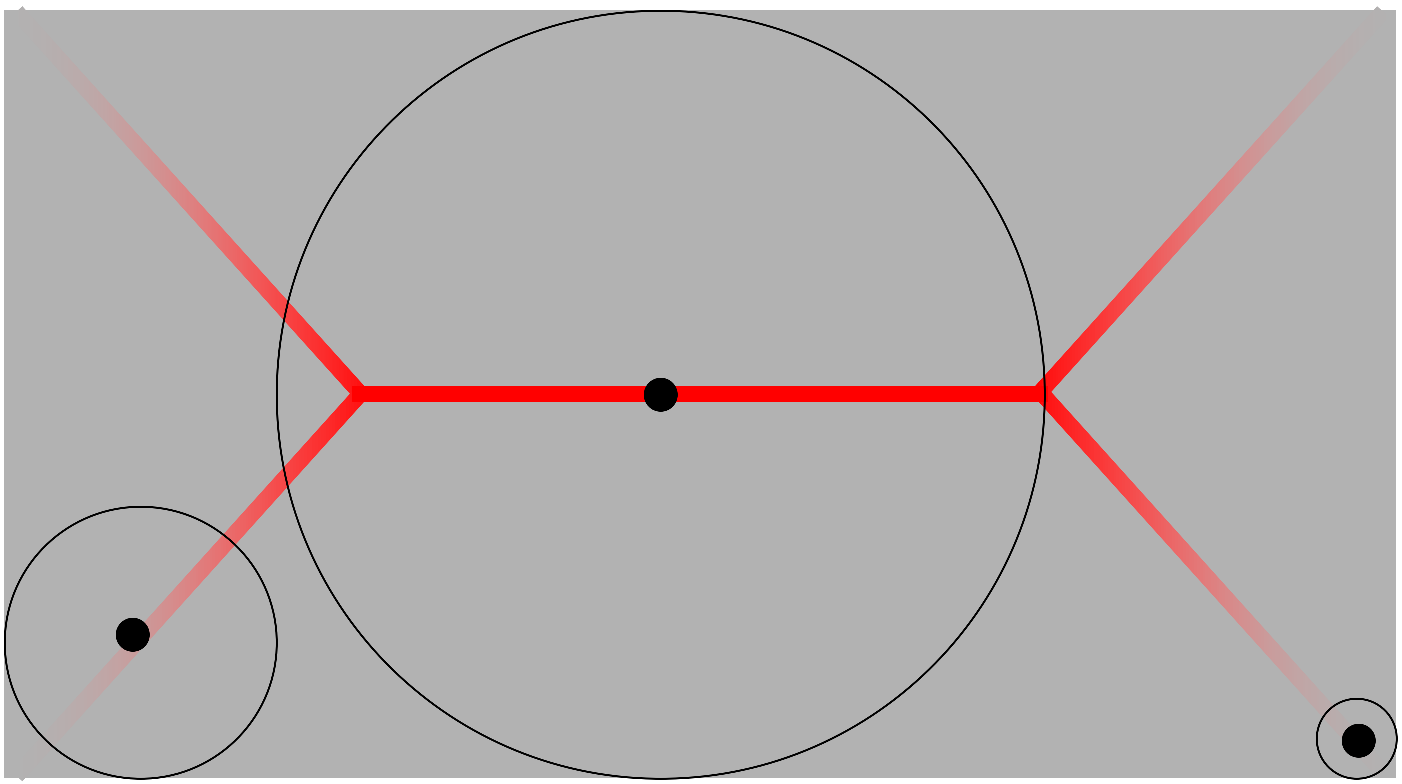


Medial axis transform

- Alternative skeleton definition:
  - The skeleton is the union of centres of maximal discs within *X*.
  - A *maximal* disc is a circular subset of *X* that touches the boundary in at least two places.

- The MAT is the skeleton with the maximal disc radius retained at each point.

Medial axis transform



Skeletonization using morphology

- Use structuring element $B =$

0	1	0
1	1	1
0	1	0

- The *n*-th skeleton subset is

$$S_n(X) = (X \ominus_n B) \setminus \left[(X \ominus_n B) \circ B \right]$$

$\ominus_n$ denotes *n* successive erosions.

- The skeleton is the union of all the skeleton subsets:

$$S(X) = \bigcup_{n=1}^{\infty} S_n(X)$$

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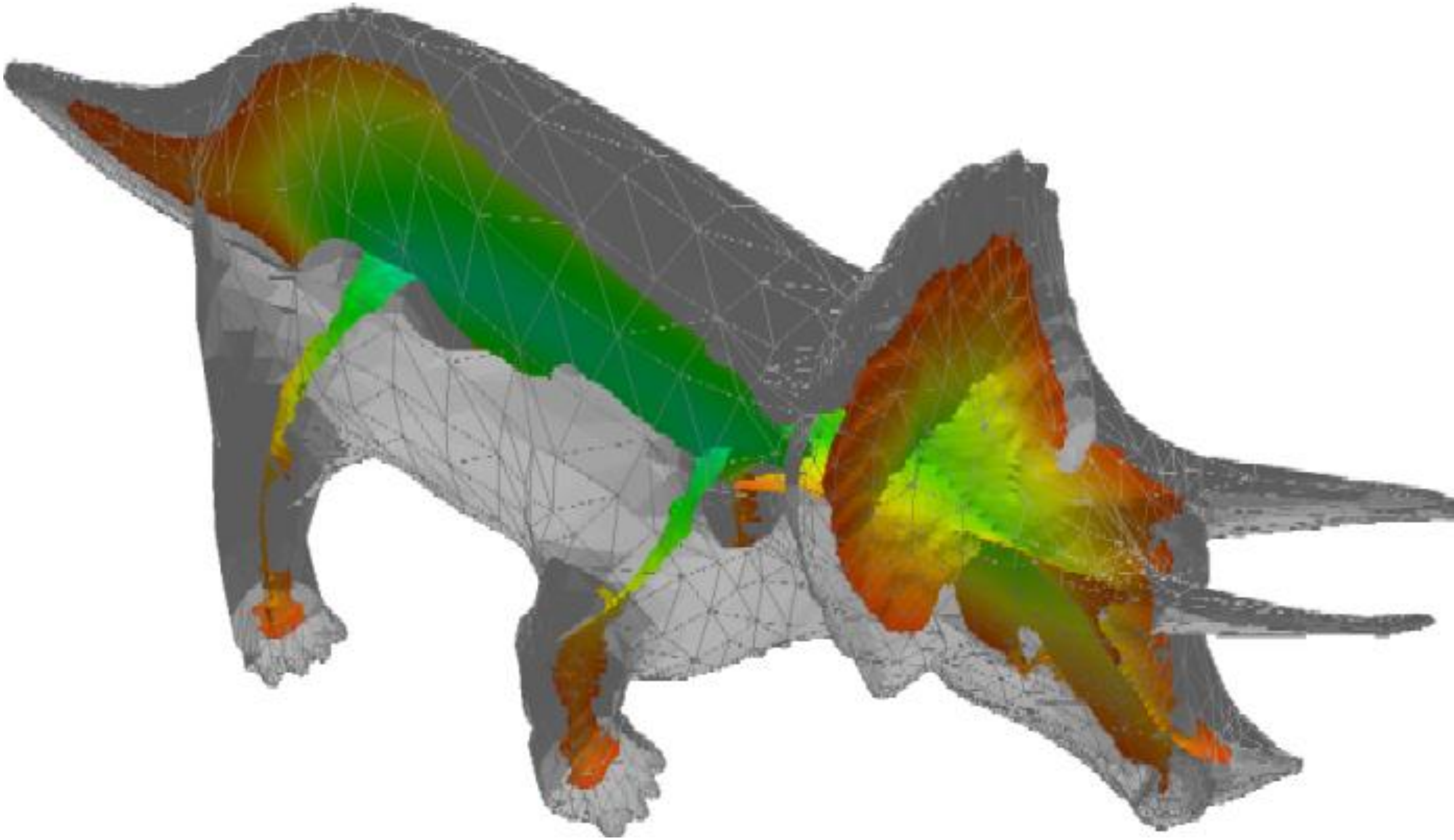
Reconstruction

- We can reconstruct region *X* from its skeleton subsets:

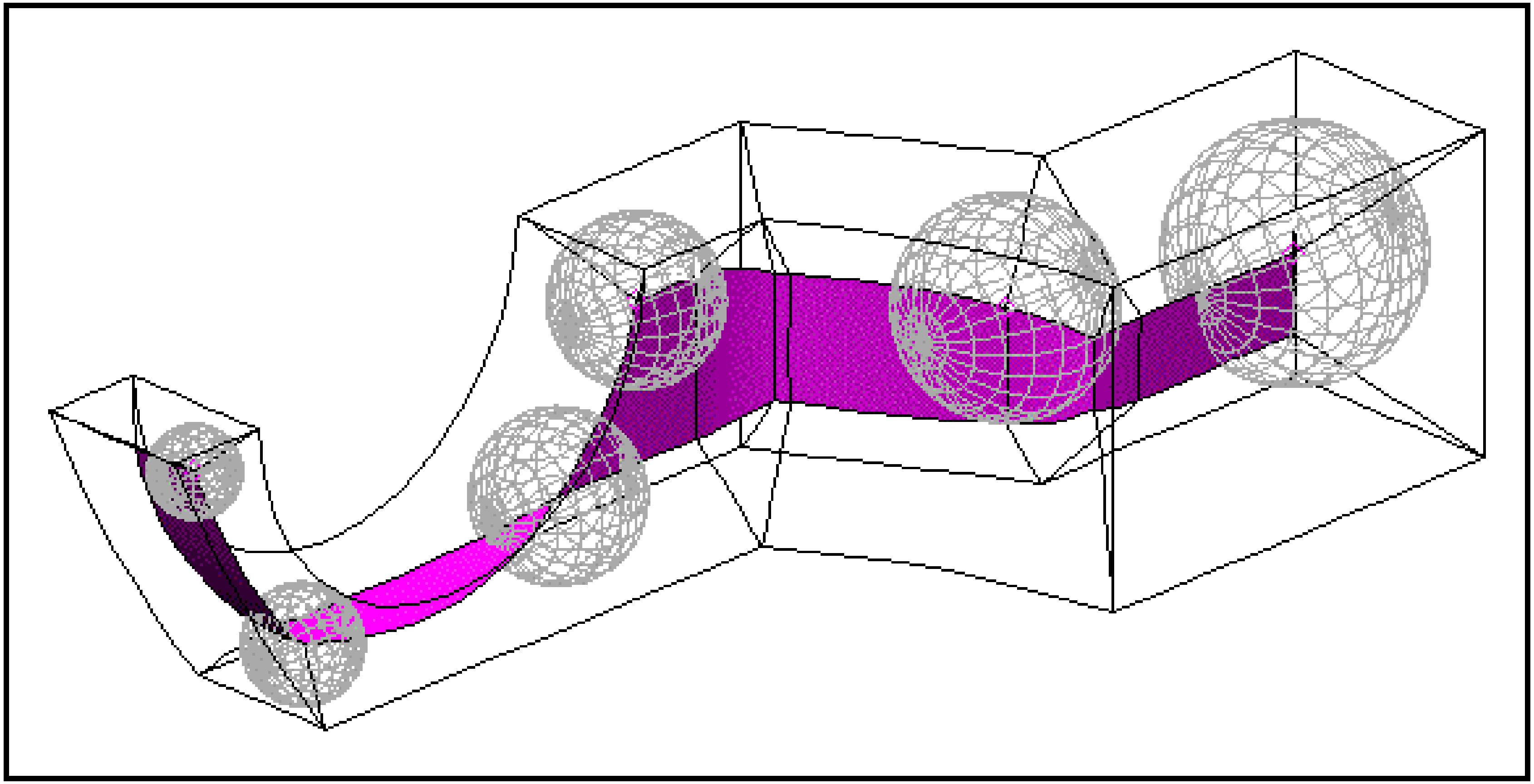
$$X = \bigcup_{n=0}^{\infty} S_n(X) \oplus_n B$$

- We can reconstruct *X* from the MAT.
- We cannot reconstruct *X* from *S*(*X*).



DiFi: Fast 3D Distance Field Computation Using Graphics Hardware
Sud, Otaduy, Manocha, Eurographics 2004

MAT in 3D



from Transcendata Europe Medial Object
Price, Stops, Butlin Transcendata Europe Ltd

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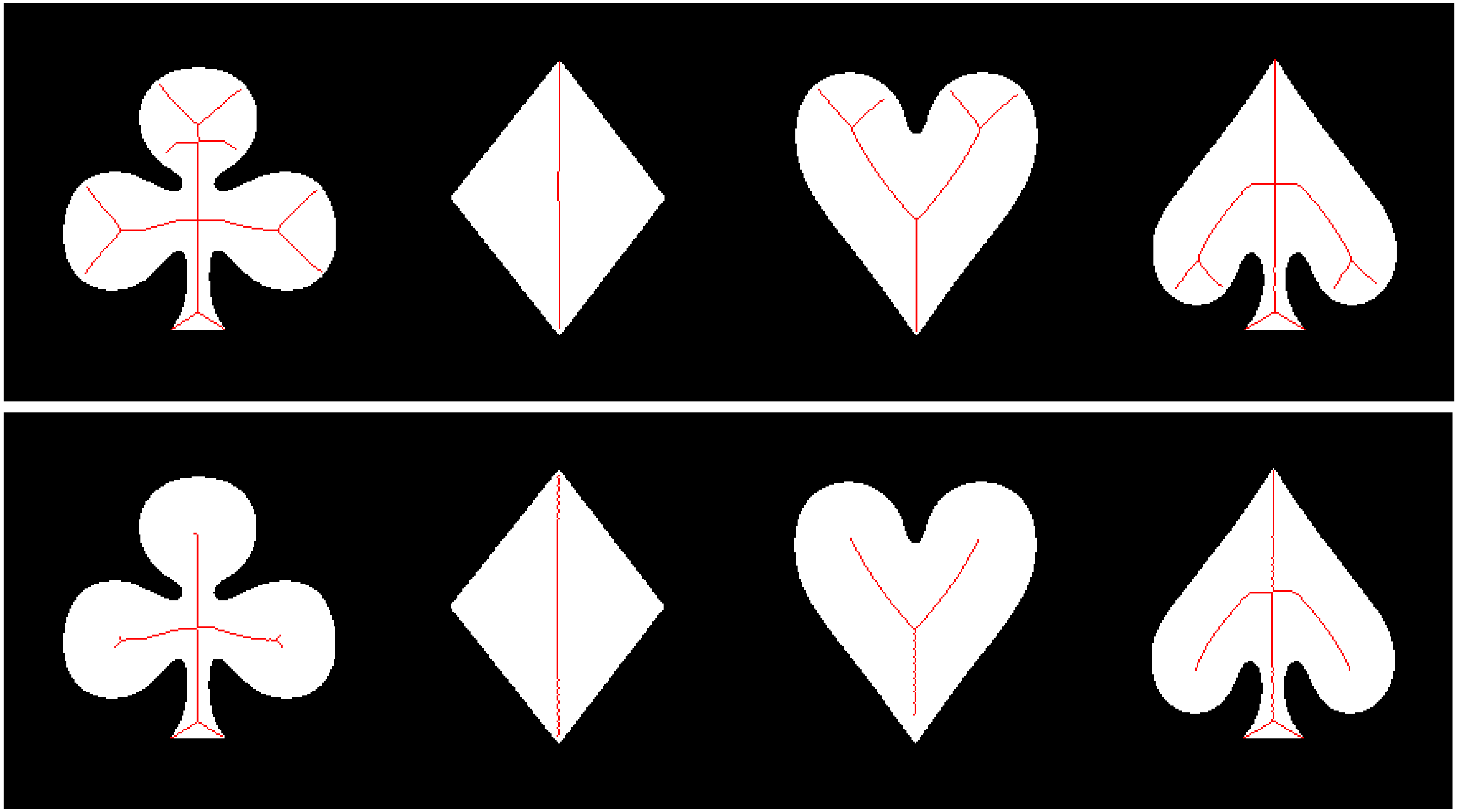
Applications and Problems

- The skeleton/MAT provides a stick figure representing the region shape
- Used in object recognition, in particular, character recognition.
- Problems:
    - Definition of a maximal disc is poorly defined on a digital grid.
    - Sensitive to noise on the boundary.
- Sequential thinning output sometimes preferred to skeleton/MAT.

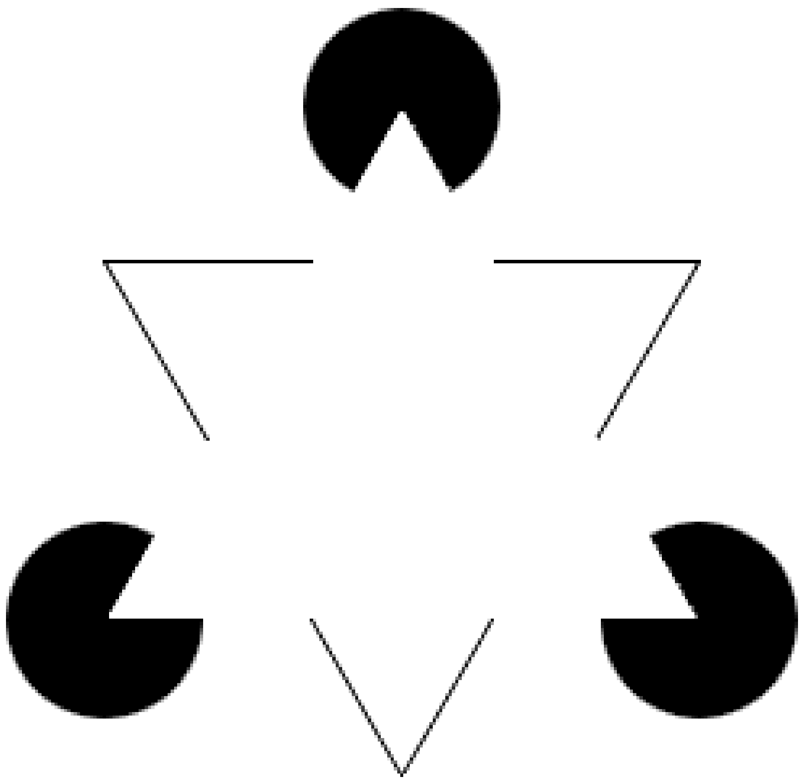
Example

Skeletons:

Thinned:



Kanizsa Triangle



Illusory Contours: Kanizsa, G. (1955), Rivista di Psicologia 49(1): 7–30

Next Week:
Image Features